Jocelyn Tran

Unit 7: Practice Problems

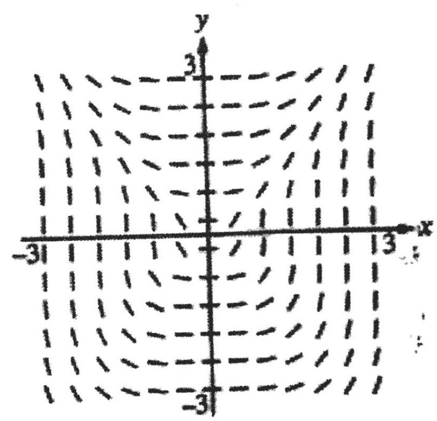
No Calculator. Show all work.

1. If $\frac{dy}{dt} = ky$ and k is a nonzero constant, then y could be

(A) $2e^{kty}$ (B) $2e^{kt}$ (C) $e^{kt} + 3$ (D) $kty + 5$ (E) $\frac{1}{2}ky^2 + \frac{1}{2}$

$y = Ce^{kt}$

2.



Shown above is a slope field for which of the following differential equations?

(A) $\frac{dy}{dx} = \frac{x}{y}$ (B) $\frac{dy}{dx} = \frac{x^2}{y^2}$ (C) $\frac{dy}{dx} = \frac{x^3}{y}$ (D) $\frac{dy}{dx} = \frac{x^2}{y}$ (E) $\frac{dy}{dx} = \frac{x^3}{y^2}$

$(\frac{1}{2}, \frac{1}{2})$

$(1,-1)$

3. If $\dfrac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

(A) $-\dfrac{2}{3}$ (B) $-\dfrac{1}{3}$ (C) 0 (D) $\dfrac{1}{3}$ (E) $\dfrac{2}{3}$

$\int \frac{1}{y^2}\,dy = \int 2\,dx$

$-y^{-1} = 2x + C$

$-\frac{1}{y} = 2x + C$

$1 = 2 + C \to C = -1$

$-y^{-1} = 2x - 1$

$y^{-1} = -2x + 1$

$y = \frac{1}{-2x+1}$

$y = \frac{1}{-3}$

4. . **The rate of change of the volume, V, of water in a tank with respect to time, t, is directly proportional to the square root of the volume. Which of the following is a differential equation that describes this relationship?**

(A) $V(t) = k\sqrt{t}$

(B) $V(t) = k\sqrt{V}$

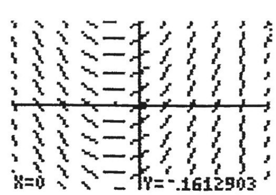
(C) $\dfrac{dV}{dt} = k\sqrt{t}$

(D) $\dfrac{dV}{dt} = \dfrac{k}{\sqrt{V}}$

(E) $\dfrac{dV}{dt} = k\sqrt{V}$

$\frac{dV}{dt} = k\sqrt{V}$

5. The slope field for a certain differentiable equation is shown below. Which of the following could be a specific solution to that differential equation?



(a) $y = 2 - x^2$ (b) $y = 3x^3 + 2$ (c) $y = 3e^{2x}$ (d) $y = x^2 + x - 3$ (e) $y = \ln x$

Calculator Allowed

6. A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present at a given time and 1000 present 2 hours later, how many will there be 5 hours from the initial given time?

(a) 2828 (b) 2143 (c) 1750 (d) 3000 (e) 16,000

$\frac{dB}{dt} = kB$

$B = Ce^{kt}$

$C = 500$

$B = 500e^{kt}$

$1000 = 500e^{2k}$

$2 = e^{2k}$ $\quad k = \frac{\ln(2)}{2}$

$\ln(2) = 2k$

(0, 500)

(2, 1000)

(5, ?)

$B = 500e^{\frac{\ln(2)}{2}t}$

$B = 500e^{\frac{\ln(2)}{2} \cdot 5}$

$B = 2828$

7. A mold culture doubles its mass every three days. Find the growth model for a plate seeded with 1.2 grams of mold if the mold grows continuously at a rate that is proportional to the amount present at any given time.

(a) $y = 1.2e^{0.23105t}$ (b) $y = 1.2e^{0.10034t}$ (c) $y = 1.2e^{0.54931t}$

(d) $y = 1.2e^{0.23856t}$ (e) $y = 1.2e^{0.69314t}$

(0, 1.2)

(3, 2.4)

$\frac{dy}{dt} = ky$

$y = Ce^{kt}$

$C = 1.2$

$y = 1.2e^{kt}$

$2.4 = 1.2e^{3k}$ $\quad y = 1.2e^{0.23105t}$

$2 = e^{3k}$

$\frac{\ln(2)}{3} = k$

8.

x	$g'(x)$
-1.0	2
-0.5	4
0.0	3
0.5	1
1.0	0
1.5	-3
2.0	-6

(-1, -2)

The table above gives selected values for the derivative of a function g on the interval $-1 \le x \le 2$. If $g(-1) = -2$ and Euler's method with a step-size of 1.5 is used to approximate $g(2)$, what is the resulting approximation?

(x, y)	$\frac{dy}{dx}$	Δx	$\Delta y = \frac{dy}{dx} \cdot \Delta x$	$(x+\Delta x, y+\Delta y)$
(-1, -2)	2	1.5	3	(0.5, 1)
(0.5, 1)	1	1.5	1.5	(2, 2.5)

(A) −6.5

(B) −1.5

(C) 1.5

(D) 2.5

(E) 3

Work out the following free response questions on the sheets provided.

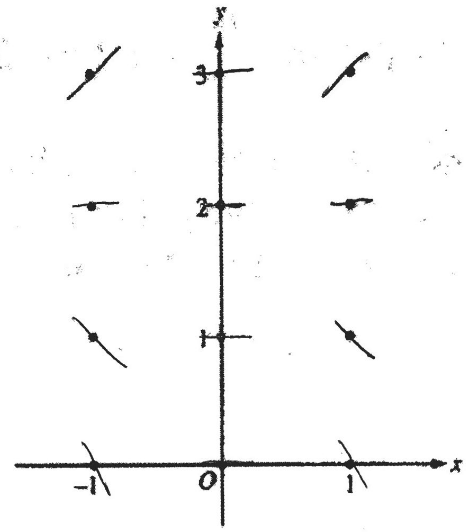
Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. **(Note: Use the axes provided in the test booklet.)**

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane. Describe all points in the *xy*-plane for which the slopes are negative.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.

Question 2a



$$\frac{dy}{dx} = x^4(y-2)$$

Question 2b

$y < 2$

$x \neq 0$

Question 2c

$$\frac{dy}{dx} = x^4(y-2)$$

$$\int \frac{1}{y-2}\,dy = \int x^4\,dx$$

$$\ln|y-2| = \frac{1}{5}x^5 + C$$

$$y-2 = Ce^{\frac{1}{5}x^5} \qquad (0,0)$$

$$-2 = C$$

$$y-2 = -2e^{\frac{1}{5}x^5}$$

$$y = -2e^{\frac{1}{5}x^5} + 2$$