Unit 2 Stations Review

General form
$A = P(1-r)^t$

Writing and Solving Equations

1. The half-life of an amount of acetaminophen in a 17 year old's body is about 45 minutes. In other words, after 45 minutes, only half of any amount of acetaminophen will remain, and the rest will have been absorbed and used. We will assume that decay is modeled by exponential decay with a constant decay rate.

   a. Suppose Janeth takes a dose of 200 mg of acetaminophen. Write an exponential formula that gives the amount of acetaminophen remaining after m half-lives.

   $A(m) = 200(0.5)^m$

   b. Adjusting your equation in part a, write an exponential formula that gives the amount of acetaminphen remaining after t minutes.

   $m = 45$
   $m = t/45$
   $A(t) = 200(0.5)^{t/45}$

   c. What is the decay rate per minute?

   $A(t) = 200(0.5)^{t/45} = 200\left[(0.5)^{1/45}\right]^t = 200(0.985)^t$

   $0.985 - 1 = -0.015 = -1.5\%$

   Keep three decimal places

   d. How much acetaminophen will remain after 100 minutes?

   $A(100) = 200(0.5)^{100/45} = 42.862$ mg

   e. How long will it take before only 30mg of acetaminophen remain?

   $30 = 200(0.5)^{t/45}$
   $\frac{30}{200} = (0.5)^{t/45}$
   $\log_{0.5}\left(\frac{30}{200}\right) = t/45$
   $45\left[\log_{0.5}\left(\frac{30}{200}\right)\right] = t = 123.16$ minutes

2. At a bank, you learn that the amount of money in your account can be modeled by the function $A = P(1 + 0.01i)^t$. P represents the principle investment, i is the interest rate, and t is in years. Solve for t in terms of A, P, and i.

   $A = P(1+0.01i)^t$
   $\frac{A}{P} = (1+0.01i)^t$
   $\log_{(1+0.01i)}\left(\frac{A}{P}\right) = t$

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Exponents and Exponent Rules

1. You conduct an experiment to measure the growth of bacteria on different media (surfaces). You find that on one agar plate, the population of bacteria at h hours is given by $B(h) = 20(2)^h$. How many bacteria are on the plate after 4 hours?

$h=4 \quad B(4)=20(2)^4 = 320$ bacteria

2. The expression $3s^2(2s^{-1} + 4s^3t)$ is equivalent to...
   a. $6s^{-2} + 12s^6t$
   b. $6s^3 + 12s^{-1}t$
   c. $8s + 7s^5t$
   d. $6s + 12s^5t$
   e. $6s^{-2} + 12st$

$6s^2s^{-1} + 12s^2s^3t$

$6s + 12s^5t$

3. The expression $(3x^2 + 2y)(3x^2 - 2y)$ is equivalent to...

Option 1: $(3x^2)(3x^2) - \cancel{2y(3x^2)} + \cancel{2y(3x^2)} + 2y(-2y)$

$9x^4 - 4y^2$

Option 2: recognize as difference of squares: $(a-b)(a+b) = a^2 - b^2$

$[(3x^2)^2 - (2y)^2] = 9x^4 - 4y^2$

Evaluate $\frac{1}{2^{-2}x^{-3}y^5}$ for $x = 2$ and $y = -4$.

Ⓐ 16 Ⓑ -4 ● $-\frac{1}{32}$ Ⓓ -16

$\frac{1}{2^{-2}(2)^{-3}(-4)^5} = \frac{2^2(2)^3}{(-4)^5} = \frac{2^5}{(-4)^5} = -\frac{1}{32}$

Simplify this expression.

$(-5g^5h^6)^2(g^4h^2)^4$

$(-5)^2 g^{10}h^{12} g^{16}h^8$

$25g^{26}h^{20}$

Rules: $(-5a)^x = -5^x a^x$

$(a^b)^c = a^{bc}$

$\frac{1}{a^{-b}} = \frac{a^b}{1}$

Answer: $25g^{26}h^{20}$

Unit 2 Stations Review

Logarithms and Solving Equations

Evaluate the following logarithms.

$\log_2(16) = 4$ $\quad 2^4 = 16$	$\log_2(\frac{1}{32}) = -5$ $\quad 2^{-5} = \frac{1}{32}$	$\log_4(2) = 1/2$ $\quad 4^{1/2} = \sqrt{4} = 2$
$\log_4(64) = 3$ $\quad 4^3 = 64$	$\log_9\left(\frac{1}{9}\right) = -1$ $\quad 9^{-1} = \frac{1}{9}$	$\log_9(3) = 1/2$ $\quad 9^{1/2} = \sqrt{9} = 3$
$\log_{\frac{1}{2}}\left(\frac{1}{8}\right) = 3$ $\quad \left(\frac{1}{2}\right)^3 = \frac{1}{8}$	$\log_{\frac{1}{2}}(64) = -6$ $\quad \left(\frac{1}{2}\right)^{-6} = 64$	$\log_{64}(4) = 1/3$ $\quad 64^{1/3} = \sqrt[3]{64} = 4$
$ln(e) = 1$ ↳ $\log_e(e)$ so What Exponent? 1	$ln(\frac{1}{e^2}) = \ln(e^{-2}) = -2$ $e^{-2} = e^{-2}$	$ln(e^{1/2}) = \frac{1}{2}\ln(e) = \frac{1}{2}$ or $e^{?} = e^{1/2} \Rightarrow \frac{1}{2}$

1. Solve each equation. Express your answer as a logarithm or exponent.

a. $4(10)^{-x} = 33$

$$10^{-x} = \frac{33}{4}$$

$$-x = \log\left(\frac{33}{4}\right)$$

$$x = -\log\left(\frac{33}{4}\right)$$

b. $100\left(5^{\frac{t}{12}}\right) = 500$

$$5^{t/12} = 5$$

$$\frac{t}{12} = \log_5(5)$$

$$\frac{t}{12} = 1$$

$$t = 12$$

c. $\log(3x + 2) = 9$

$$10^9 = 3x + 2$$

$$10^9 - 2 = 3x$$

$$\frac{10^9 - 2}{3} = x$$

Unit 2 Stations Review

Average Rate of Change: Slope!

$\frac{y_2 - y_1}{x_2 - x_1}$

Average Rate of Change and Geometric Series

1. A social media site is experiencing rapid growth. The table below shows values of V, the total number of unique visitors in each month, t, for a 6-month period of time. The graph shows the average minutes per visit to the site, M, in each month, t, for the same 6-month period of time.

t, Month	1	2	3	4	5	6
$V(t)$, Number of Unique Visitors	418,000	608,000	1,031,000	1,270,000	2,023,000	3,295,000

a. What is the average rate of change of unique visitors between months 2 and 5?

(2, 608000)

(5, 2023000)

$\frac{2023000 - 608000 \text{ visitors}}{5 - 2 \text{ months}}$ = 471666.667 visitors/month

b. Between what two months did the site experience the largest user growth? Justify your answer.

Between months 5 and 6, the site had the most user growth.

3295000 − 2023000 = 1272000 visitors

↳ increase is > 1 million

only month where this happens

2. For each series below, identify the following

   I. Is the series a geometric series? → means there is a common factor/ratio: times -3, divide by 2, etc.

   II. If it is a geometric series, what is its common ratio?

   III. Write a formula that would give us the nth term in the sequence. Assume the first term in n=1.

a) 2, 10, 50, 250, …

$\frac{10}{2} = 5$

$\frac{50}{10} = 5$

$\frac{250}{50} = 5$

i. geometric series

ii. common ratio is 5

iii. $f(n) = 2(5)^{n-1}$

b) 3, 5, 7, 9, 11,…

$\frac{5}{3} = 1$ and $2/3$

$\frac{7}{5} = 1$ an $2/5$

not a common ratio!

Not geometric.

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Graphs and Transformations

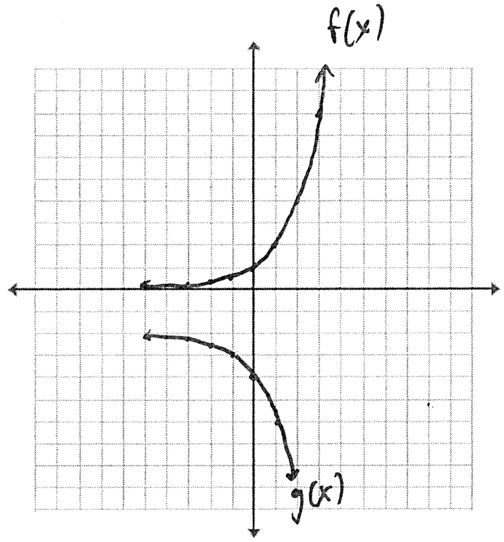
For each set of functions, graph f and g. Then state what transformations take the graph of f and transform it into g.

$f(x)=2^x$, and $g(x)=-2^{x+1}-2$

x	2^x
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

Reflect over x-axis
Down 2
Left 1

$\longrightarrow$ g(x)



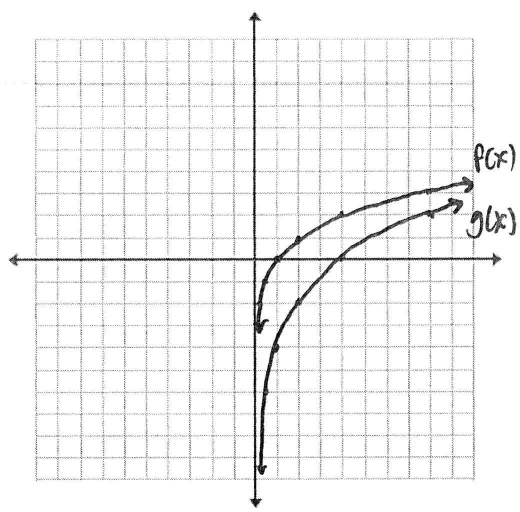
$f(x)=\log_2(x)$, and $g(x)=2\log_2(x)-4$

x	$\log_2(x)$
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

Vertical Scaling by a factor of 2

Down 4

$\longrightarrow$



Remember, Vertical scaling means take the y-value and multiply it by ______. in this case: 2.