Name ____________________ Date ____________

1. Use properties of exponents to rewrite each expression with only positive, rational exponents. Then find the numerical value of each expression when $x = 9$, $y = 8$, and $z = 16$. In each case, the expression evaluates to a rational number.

   a. $\sqrt{\dfrac{xy^2}{(x^4z)^{\frac{1}{2}}}} = \dfrac{x^{1/2}y}{\sqrt{x^2z}^{1/2}} = \dfrac{x^{1/2}y}{xz^{1/4}} = \dfrac{\sqrt{x}\,y}{x\sqrt[4]{z}}$

   $= \dfrac{\sqrt{9}\,(8)}{9\sqrt[4]{16}} = \dfrac{3\cdot 8}{9\cdot 2} = \dfrac{24}{18} = \dfrac{12}{9} = \dfrac{4}{3}$

   b. $\dfrac{\cancel{x}y^3z^{-1}}{\cancel{x}y^2z^{-2}} = \dfrac{yz^{-1}}{z^{-2}} = yz^{-1}z^{2} = yz = 8(16) = 128$

2.

Determine if the sequence is geometric. If it is, find the common ratio.

1) $-1, 6, -36, 216, \ldots$

$\frac{6}{-1} = -6$

$\frac{-36}{6} = -6$

$\frac{216}{-36} = -6$

Geometric sequence ratio is -6

2) $-1, 1, 4, 8, \ldots$

$\times -1 \quad \times 4 \quad \times 2$

Not geometric

3. Use the properties of exponents to identify the percent rate of change of the functions below, and classify them as representing exponential growth or decay. Afterward, identify the common factor.

Get into basic $A = P(1-r)^t$ form.

   a. $f(t) = (1.72)^t$

   $1.72 - 1 = 0.72$

   $r = 72\%$ growth common factor: 1.72

   b. $(t) = (1.01)^{3t}$

   $[(1.01)^3]^t = (1.030301)^t$

   $1.030301 - 1 = 0.030301$ $r = 3.03\%$ growth

   common factor: 1.030301

   c. $f(t) = 3(0.84)^t$

   $0.84 - 1 = -0.16$

   $r = -16\%$ decay

   common factor: 0.84

4. A scientist is studying the growth of a population of bacteria. At the beginning of her study, she has 800 bacteria. She notices that the population is quadrupling every hour.
   x4

   a. What quantities, including units, need to be identified to further investigate the growth of this bacteria population? Explain your reasoning.

   Does the population keep quadrupling consistently?
   So we need the rate of change to be consistent.

   b. The scientist recorded the following information in her notebook, but she forgot to label each row. Label each row to show what quantities, **including appropriate units**, are represented by the numbers in the table, and then complete the table.

t (hours)	0	1	2	3	4
Bacteria (hundreds)	8	32	128		

   c. Write an explicit formula for the number of bacteria present after t hours.

   $B(t) = 800(4)^t$

   d. Another scientist studying the same population notices that the population is doubling every half an hour. Complete the table, and write an explicit formula for the number of bacteria present after x half hours.

Time, t (hours)	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3
Time, x (half-hours)	0	1	2	3	4	5	6
Bacteria (hundreds)	8	16	32	64	128	256	512

   $B(x) = 800(2)^x$

5. Solve each equation. Express your answer as a logarithm or exponent.

a. $3(10)^{-x} = \frac{1}{9}$

$10^{-x} = \frac{1}{27}$

$-x = \log\left(\frac{1}{27}\right)$

$x = -\log\left(\frac{1}{27}\right)$

b. $362\left(10^{\frac{t}{12}}\right) = 500$

$10^{t/12} = \frac{500}{362}$

$\frac{t}{12} = \log\left(\frac{500}{362}\right)$

$t = 12\log\left(\frac{500}{362}\right)$

c. $\log(3x - 4) = 9$

$3x - 4 = 10^9$

$3x = 10^9 + 4$

$x = \frac{10^9 + 4}{3}$

d. $300\ln(2x) + 600 = 900$

$300\ln(2x) = 300$

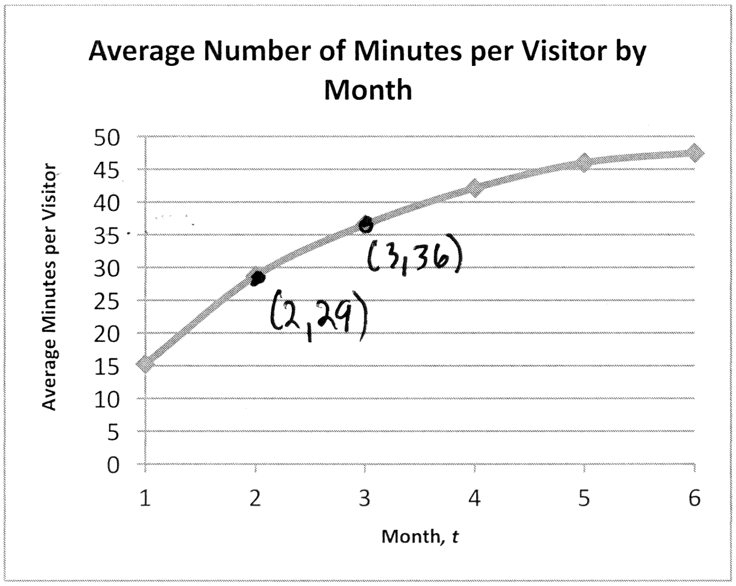
$\ln(2x) = 1$

$2x = e^1$

$x = \frac{e}{2}$

6. A social media site is experiencing rapid growth. The table below shows values of V, the total number of unique visitors in each month, t, for a 6-month period of time. The graph shows the average minutes per visit to the site, M, in each month, t, for the same 6-month period of time.

t, Month	1	2	3	4	5	6
$V(t)$, Number of Unique Visitors	418,000	608,000	1,031,000	1,270,000	2,023,000	3,295,000



a. Between which two months did the site experience the most growth in total unique visitors? What is the average rate of change over this time interval?

from $t=5$, 2023000 to $t=6$, 3295000

$\frac{3295000-2023000}{6-5} = 1272000$ visitors/month

b. Compute the value of $\frac{V(6)-V(1)}{6-1}$, and explain its meaning in this situation.

(6, 3295000)
(1, 418000)

$\frac{3295000-418000}{6-1} = 575400$ visitors/month

average increase in visitors to the site/month

c. Estimate the value of $\frac{M(3)-M(2)}{3-2}$ from the graph of M, and explain its meaning in this situation.

(3, 36)
(2, 29)

$\frac{36-29 \text{ minutes}}{3-2 \text{ months}} = 7$ min/month

average increase in site use in minutes per month

7. Each function below models the growth of two different trees of different ages over a fixed time interval.

Tree A:

$f(t) = 12(1.21)^{\frac{t}{2}}$, where t is time in years since the tree was 12 feet tall, $f(t)$ is the height of the tree in feet, and $0 \leq t \leq 4$.

Tree B:

Years since the tree was 4 feet tall, t	Height in feet after t years, $g(t)$
0	4
1	4.8
2	5.76
3	6.912
4	8.2944

$4.8/4 = 1.2 \rightarrow 20\%$
$5.76/4.8 = 1.2 \rightarrow 20\%$

a. Use the properties of exponents to show that Tree A has a percent rate of change of 10% per year.

$f(t) = 12(1.21)^{1/2 \, t} = 12(1.10)^t$

$r = 10\%$

b. Which tree, A or B, has the greatest percent rate of change per year? Justify your answer.

Tree A: 10% per year

Tree B: 20% per year

Tree B's is greater

c. When will Tree A be 20 feet tall? Round your answer to the nearest hundredths and include units.

$t = ?$

$20 = 12(1.21)^{t/2}$

$\frac{20}{12} = (1.21)^{t/2}$

$\log_{1.21}\left(\frac{20}{12}\right) = \frac{t}{2}$

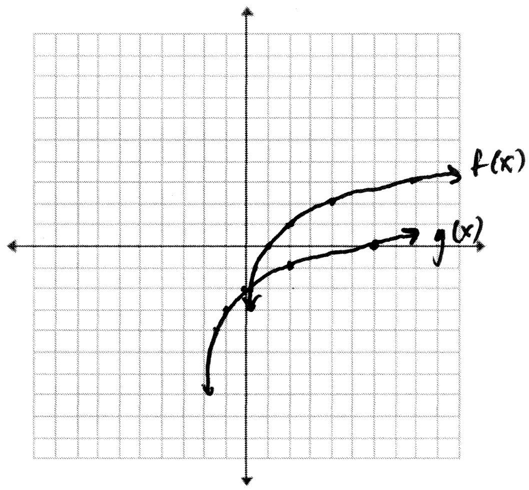
$2\log_{1.21}\left(\frac{20}{12}\right) = t = \boxed{5.360 \text{ years}}$

1. For parts (a) to (c),
   - Sketch the graph of each pair of functions on the same coordinate axes showing end behavior and intercepts, and
   - Describe the graph of g as a series of transformations of the graph of f.

$f(x)=\log_2(x)$, and $g(x)=\log_2(x+2)-3$

x	$\log_2(x)$
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3

Down 3
left 2
→ g(x)



In the question above, what is the domain of f(x) and g(x)?

f(x): x > 0
g(x): x > -2 b/c g(x) is shifted left 2

1. For parts (a) to (c),
    - Sketch the graph of each pair of functions on the same coordinate axes showing end behavior and intercepts, and
    - Describe the graph of g as a series of transformations of the graph of f.

    a. $f(x) = 2^x$, and $g(x) = 2^{-x} + 3$

x	f(x)
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

reflect over y
up 3

g(x)