

Practice MC

1. $A = \pi r^2$ $dA = 2\pi r dr$ we want $dA = 2 dr$ so $\pi r = 1$

$$r = \frac{1}{\pi}$$

2. Know
 $\frac{dr}{dt} = 0.3 \frac{m}{sec}$

Want to know
 $\frac{dV}{dt}$ when $S = 100\pi$

Egn
 $V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 100\pi(0.3)$$

$\frac{dV}{dt} = 30\pi E$

3. Know
 $\frac{dr}{dt} = \frac{1}{2}$
 $\frac{dh}{dt} = \frac{1}{2}$

Want to know
 $\frac{dV}{dt}$ when $h = 9 \text{ cm}$
 $r = 6 \text{ cm}$

Egn
 $V = \frac{1}{3}\pi r^2 h$

$$\frac{dV}{dt} = \frac{1}{3}\pi \left[r^2 \frac{dh}{dt} + h(2r) \frac{dr}{dt} \right]$$

$$\frac{dV}{dt} = \frac{1}{3}\pi \left[36\left(\frac{1}{2}\right) + 18(6)\left(\frac{1}{2}\right) \right] = \frac{1}{3}\pi [18 + 54]$$

$= \frac{1}{3}\pi(72) = 24\pi C$

4. Know
 $\frac{dz}{dt} = 1$
 $\frac{dx}{dt} = 3 \frac{dy}{dt}$
 so $\frac{dy}{dt} = \frac{1}{3} \frac{dx}{dt}$

want to know
 $\frac{dx}{dt}$ when $x=4$
 $y=3$
 $4^2 + 3^2 = 5^2$

egn

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$4 \left(\frac{dx}{dt} \right) + 3 \left(\frac{1}{3} \frac{dx}{dt} \right) = 5(1)$$

$5 \frac{dx}{dt} = 5 \quad \left[\frac{dx}{dt} = 1 \right] B$

5. Know
 $\frac{ds}{dt} = \frac{4}{9}$

Want to know
 $\frac{dx}{dt}$ when

Egn.

$$s = \frac{s+x}{2}$$

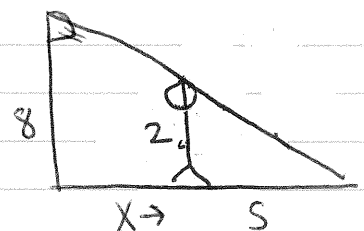
$$8s = 2s + 2x$$

$$4s = s + x$$

$$3s = x$$

$$3 \frac{ds}{dt} = \frac{dx}{dt}$$

$\frac{dx}{dt} = 3 \left(\frac{4}{9} \right) = \frac{4}{3} D$



6.

Know

$$\frac{dh}{dt} = -3$$

Want to know

$$\frac{dx}{dt} \text{ when } h=7$$

$$\begin{aligned} 7^2 + x^2 &= 25^2 \\ 49 + x^2 &= 625 \\ x^2 &= 576 \\ \underline{x=24} \end{aligned}$$

$$\begin{array}{r} 25 \\ 25 \\ \hline 125 \\ 500 \end{array}$$

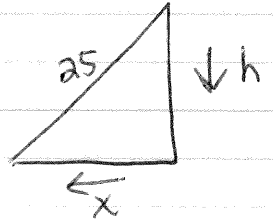
$$\begin{array}{r} 51 \\ 49 \\ \hline 576 \end{array}$$

Egn + Picture

$$\begin{aligned} x^2 + h^2 &= 25^2 \\ 2x \frac{dx}{dt} + 2h \frac{dh}{dt} &= 0 \end{aligned}$$

$$\frac{dx}{dt} = \frac{-2h \frac{dh}{dt}}{2x}$$

$$\frac{dx}{dt} = \frac{-7}{24} (-3) = \frac{21}{24} = \boxed{\frac{7}{8} \text{ ft/min}} \quad \text{W}$$



7.

Know

?

Egns

$$A = \pi r^2$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$C = 2\pi r$$

$$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$$

Want to know

$$r \text{ when } \frac{dA}{dt} = \frac{dC}{dt}$$

$$\frac{dA}{dt} = \frac{dC}{dt}$$

$$2\pi r \frac{dr}{dt} = 2\pi \frac{dr}{dt}$$

$$r = \frac{2\pi r \frac{dr}{dt}}{2\pi \frac{dr}{dt}}$$

$$\boxed{r=1} \quad \text{D}$$

8.

Know

$$\frac{dy}{dt} < 0$$

Egns

$$\begin{aligned} x^2 + y^2 &= 40^2 \\ 2x \frac{dx}{dt} + 2y \frac{dy}{dt} &= 0 \end{aligned}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = 0$$

$$x \left(\frac{3}{4} \frac{dy}{dt} \right) + y \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} \left(y - \frac{3}{4}x \right) = 0$$

$$\text{so } \frac{dy}{dt} = 0 \text{ or}$$

$$y - \frac{3}{4}x = 0 \quad y = \frac{3}{4}x$$

Want to know

$$x \text{ when } \frac{dx}{dt} = \frac{3}{4} \left(\frac{dy}{dt} \right)$$

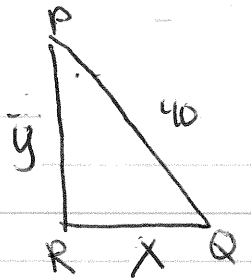
$$x^2 + \left(\frac{3}{4}x \right)^2 = 40^2$$

$$x^2 + \frac{9}{16}x^2 = 40^2$$

$$\frac{25}{16}x^2 = 40^2$$

$$\frac{5}{4}x = 40$$

$$x = 40 \left(\frac{4}{5} \right) = \boxed{32}$$



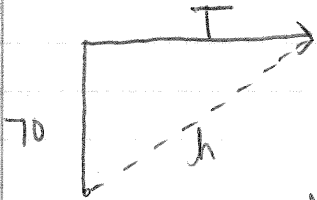
9.

Know

$$\frac{dT}{dt} = 60 \frac{m}{s}$$

Want to know

$$\frac{dh}{dt} \text{ when } T = 60(4) = 240$$



Egn

$$70^2 + T^2 = h^2$$

$$0 + 2T \frac{dT}{dt} = 2h \frac{dh}{dt}$$

$$h = \sqrt{70^2 + 240^2} = 250$$

$$\frac{dh}{dt} = \frac{T \frac{dT}{dt}}{h} = \frac{240(60)}{250}$$

$$\frac{dh}{dt} = 57.6 \text{ m/s } \boxed{A}$$

10.

Know

$$\frac{d\theta}{dt} = \frac{3 \text{ rads}}{\text{min}}$$

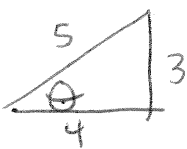


Want to know

$$\frac{dx}{dt} \text{ when } x=3$$

Egns

$$\sin \theta = \frac{x}{5} \quad x = 5 \sin \theta$$



$$\frac{dx}{dt} = 5 \cos \theta \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = 5 \left(\frac{4}{5} \right) \left(3 \right) = \boxed{12 \text{ E}}$$

11.

Know

$$\frac{dV}{dt} = 24$$

$$\frac{ds}{dt} = 2$$

Want to know

s

Egn

$$V = s^3 \quad \frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$24 = 3s^2(2)$$

$$4 = s^2 \quad \boxed{s=2 \text{ A}}$$

12.

Know

$$\frac{dV}{dt} = 24$$

$$\frac{dA}{dt} = 12$$

Want to know

s

Egn

$$V = s^3$$

$$A = 6s^2$$

$$\frac{dV}{dt} = 3s^2 \frac{ds}{dt}$$

$$\frac{dA}{dt} = 12s \frac{ds}{dt}$$

$$24 = 3s^2 \frac{ds}{dt}$$

$$12 = 12s \frac{ds}{dt}$$

$$\boxed{E} \quad \boxed{s=8}$$

$$\leftarrow \frac{8}{s^2} = \frac{1}{s}$$

$$\frac{8}{s^2} = \frac{ds}{dt}$$

$$\frac{1}{s} = \frac{ds}{dt}$$