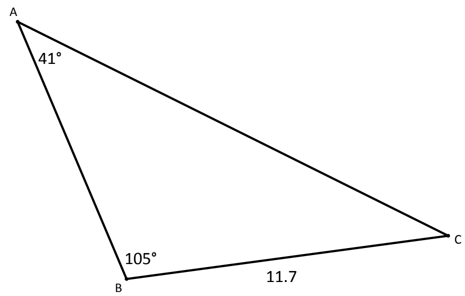
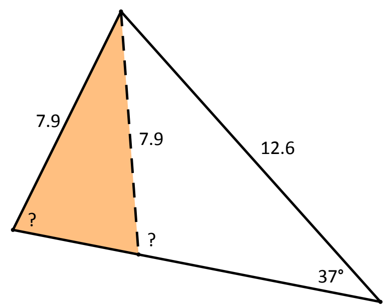
Exit Ticket Sample Solutions

1. Find the length of side $\overline{AC}$ in the triangle below.



We have $\frac{AC}{\sin(105^\circ)} = \frac{11.7}{\sin(41^\circ)}$*, which gives* $C = \frac{11.7 \cdot \sin(105^\circ)}{\sin(41^\circ)} \approx 17.2$ *.*

2. A triangle has sides with lengths 12.6 and 7.9. The angle opposite 7.9 is $37°$. What are the possible values of the measure of the angle opposite 12.6?



We have $\frac{\sin(x)}{12.6} = \frac{\sin(37^\circ)}{7.9}$*, which means that* $\sin(x) = \frac{12.6 \cdot \sin(37^\circ)}{7.9}$*. The values of* x *that satisfy this equation are* $x \approx 73.7^\circ$ *and* $x \approx 106.3^\circ$*.*

Problem Set Sample Solutions

1. Let $\triangle ABC$ be a triangle with the given lengths and angle measurements. Find all possible missing measurements using the law of sines.

   a. $a = 5, m\angle A = 43^\circ, m\angle B = 80^\circ$.

   $b \approx 7.22, c \approx 6.15, m\angle C = 57^\circ$

   b. $a = 3.2, m\angle A = 110^\circ, m\angle B = 35^\circ$.

   $b \approx 1.95, c \approx 1.95, m\angle C = 35^\circ$

   c. $a = 9.1, m\angle A = 70^\circ, m\angle B = 95^\circ$.

   $b \approx 9.65, c \approx 2.51, m\angle C = 15^\circ$

d. $a = 3.2, m\angle B = 30°, m\angle C = 45°$.

$m\angle A = 105°, b \approx 1.66, c \approx 2.34$

e. $a = 12, m\angle B = 29°, m\angle C = 31°$.

$m\angle A = 120°, b \approx 6.72, c \approx 7.14$

f. $a = 4.7, m\angle B = 18.8°, m\angle C = 72°$.

$m\angle A = 89.2°, b \approx 1.51, c \approx 4.47$

g. $a = 6, b = 3, m\angle A = 91°$.

$m\angle B \approx 29.99°, m\angle C \approx 59.01°, c \approx 5.14$

h. $a = 7.1, b = 7, m\angle A = 70°$.

$m\angle B = 67.89°, m\angle C \approx 42.11°, c = 5.07$

i. $a = 8, b = 5, m\angle A = 45°$.

$m\angle B = 26.23°, m\angle C = 108.77°, c = 10.71$

j. $a = 3.5, b = 3.6, m\angle A = 37°$.

$m\angle B = 38.24°, m\angle C = 104.76°, c = 5.62$ *or* $m\angle B = 141.76°, m\angle C = 1.24°, c = 0.13$

k. $a = 9, b = 10.1, m\angle A = 61°$.

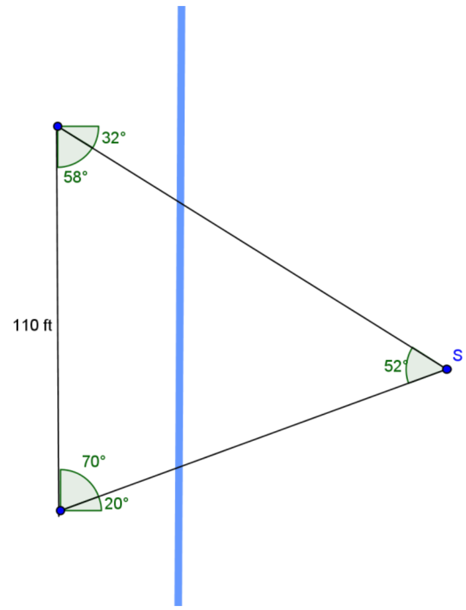
$m\angle B = 78.97°, m\angle C = 40.03°, c = 6.62$ *or* $m\angle B = 101.03°, m\angle C = 17.97°, c = 3.17$

l. $a = 6, b = 8, m\angle A = 41.5°$.

$m\angle B = 62.07°, m\angle C = 76.43°, c = 8.8$ *or* $m\angle B = 117.93°, m\angle C = 20.57°, c = 3.18$

2. A surveyor is working at a river that flows north to south. From her starting point, she sees a location across the river that is $20°$ north of east from her current position, she labels the position S. She moves 110 feet north and measures the angle to S from her new position, seeing that it is $32°$ south of east.

a. Draw a picture representing this situation.



b. Find the distance from her starting position to S.

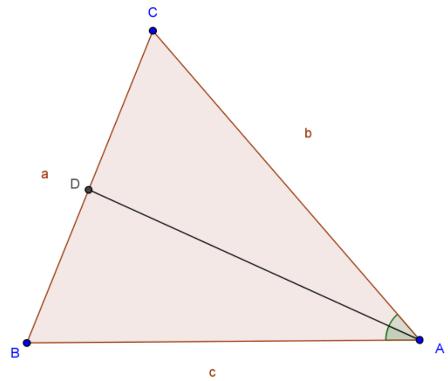
$$\frac{\sin(52°)}{110} = \frac{\sin(58°)}{x}$$
$$x \approx 118.4$$

The distance is about 118 ft.

c. Explain how you can use the procedure the surveyor used in this problem (called triangulation) to calculate the distance to another object.

Calculate the angle from the starting point to the object. Travel a distance from the starting point, and again calculate the angle to the object. Create a triangle connecting the starting point, the object, and the new location. Use that distance traveled and the angles to find the distance to the object.

3. Consider the triangle pictured below.



Use the law of sines to prove the generalized angle bisector theorem, that is, $\frac{BD}{DC} = \frac{c\sin(m\angle BAD)}{b\sin(m\angle CAD)}$. (Although this is called the generalized angle bisector theorem, we do not assume that the bisector of $\angle BAC$ intersects side $\overline{BC}$ at D. In the case that $\overline{AD}$ is an angle bisector, then the formula simplifies to $\frac{BD}{DC} = \frac{c}{b}$.)

a. Use the triangles ABD and ACD to express $\frac{c}{BD}$ and $\frac{b}{DC}$ as a ratio of sines.

$$\frac{c}{BD} = \frac{\sin(m\angle BDA)}{\sin(m\angle BAD)}$$

$$\frac{b}{DC} = \frac{\sin(m\angle CDA)}{\sin(m\angle CAD)}$$

b. Note that angles BDA and ADC form a linear pair. What does this tell you about the value of the sines of these angles?

Since the angles are supplementary, the sines of these values are equal.

c. Solve each equation in part (a) to be equal to the sine of either $\angle BDA$ or $\angle ADC$.

$$\sin(m\angle BAD) \cdot \frac{c}{BD} = \sin(m\angle BDA)$$

$$\sin(m\angle CAD) \cdot \frac{b}{DC} = \sin(m\angle CDA)$$

d. What do your answers to parts (b) and (c) tell you?

The answers tell me that the two equations written in part (c) are equal to each other.

e. Prove the generalized angle bisector theorem.

From part (d), we have

$$\sin(m\angle BAD) \cdot \frac{c}{BD} = \sin(m\angle CAD) \cdot \frac{b}{DC}$$

Dividing both sides by $\sin(m\angle CAD) \cdot b$, and multiplying by BD, we get

$$\frac{BD}{DC} = \frac{c\sin(m\angle BAD)}{b\sin(m\angle CAD)}.$$

4. As an experiment, Carrie wants to independently confirm the distance to Alpha Centauri. She knows that if she measures the angle of Alpha Centauri and waits 6 months and measures again, then she will have formed a massive triangle with two angles and the side between them being 2 AU long.

a. Carrie measures the first angle at $82°\ 8'24.5''$ and the second at $97°\ 51'34''$. How far away is Alpha Centauri according to Carrie's measurements?

The third angle would be $1.5''$.

$$\frac{\sin\left(82+\frac{8}{60}+\frac{24.5}{3600}\right)}{a}=\frac{\sin\left(\frac{1.5}{3600}\right)}{2}$$

$$a \approx 272\,436$$

About $272{,}436$ AU.

b. Today, astronomers use the same triangulation method on a much larger scale by finding the distance between different spacecraft using radio signals and then measuring the angles to stars. Voyager 1 is about 122 AU away from Earth. What fraction of the distance from Earth to Alpha Centauri is this? Do you think that measurements found in this manner are very precise?

Voyager 1 is about $\frac{122}{276\,364}$, or 0.0004 the distance of Earth to Alpha Centauri. Depending on how far away the object being measured is, the distances are fairly precise on an astronomical scale. One AU is almost 93 million miles, which is not very precise.

5. A triangular room has sides of length 3.8 m, 5.1 m, and 5.1 m. What is the area of the room?

Since the room is isosceles, the height bisects the side of length 3.8 at a right angle. We get $\cos(\theta)=\frac{1.9}{5.1}$; therefore, $\theta \approx 68.127°$.

$$\frac{1}{2}\cdot 3.8\cdot 5.1\cdot \sin(68.127°)\approx 8.99$$

The area of the room is approximately $8.99\ \text{m}^2$.