Answers for Problem Set 6.3

$$\cos\left(-\frac{7\pi}{12}\right)$$

$$\begin{aligned}\cos\left(-\frac{7\pi}{12}\right) &= \cos\left(-\frac{\pi}{4}-\frac{\pi}{3}\right)\\ &= \cos\left(-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(-\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\frac{1}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

$$\sin\left(\frac{13\pi}{12}\right)$$

$$\begin{aligned}\sin\left(\frac{13\pi}{12}\right) &= \sin\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)\\ &= \sin\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\cos\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\frac{1}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

$$\cos\left(-\frac{13\pi}{12}\right)$$

$$\begin{aligned}\cos\left(-\frac{13\pi}{12}\right) &= \cos\left(-\frac{\pi}{4}-\frac{\pi}{3}\right)\\ &= \cos\left(-\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(-\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)\\ &= \frac{\sqrt{2}}{2}\cdot\frac{1}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$

$$\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{12}\right)$$

$$\begin{aligned}\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{6}\right)\\ &= \frac{1}{2}\end{aligned}$$

$$\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{6}\right)-\cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right)$$

$$\begin{aligned}\sin\left(\frac{5\pi}{12}\right)\cos\left(\frac{\pi}{6}\right)-\cos\left(\frac{5\pi}{12}\right)\sin\left(\frac{\pi}{6}\right) &= \sin\left(\frac{\pi}{4}\right)\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

Answers for Problem Set 6.3

$\tan\left(\frac{\pi}{12}\right)$

$$\tan\left(\frac{\pi}{12}\right)=\tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right)=\frac{\tan\left(\frac{\pi}{3}\right)-\tan\left(\frac{\pi}{4}\right)}{1+\tan\left(\frac{\pi}{3}\right)\tan\left(\frac{\pi}{4}\right)}=\frac{\sqrt{3}-1}{1+\sqrt{3}}=2-\sqrt{3}$$

$\tan\left(-\frac{\pi}{12}\right)$

$$\tan\left(-\frac{\pi}{12}\right)=\tan\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=\frac{\tan\left(\frac{\pi}{4}\right)-\tan\left(\frac{\pi}{3}\right)}{1+\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}=\frac{1-\sqrt{3}}{1+\sqrt{3}}=-2+\sqrt{3}$$

$\tan\left(\frac{7\pi}{12}\right)$

$$\tan\left(\frac{7\pi}{12}\right)=\tan\left(\frac{\pi}{4}+\frac{\pi}{3}\right)=\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{3}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}=\frac{1+\sqrt{3}}{1-\sqrt{3}}=-2-\sqrt{3}$$

$\tan\left(-\frac{13\pi}{12}\right)$

$$\tan\left(-\frac{13\pi}{12}\right)=\tan\left(\frac{\pi}{4}-\frac{4\pi}{3}\right)=\frac{\tan\left(\frac{\pi}{4}\right)-\tan\left(\frac{4\pi}{3}\right)}{1+\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{4\pi}{3}\right)}=\frac{1-\sqrt{3}}{1+\sqrt{3}}=-2+\sqrt{3}$$

$\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{12}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{12}\right)}$

$$\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{12}\right)}{1-\tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{12}\right)}=\tan\left(\frac{\pi}{4}+\frac{\pi}{12}\right)=\tan\left(\frac{4\pi}{12}\right)=\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$$

Use the sum formula for sine to show that $\sin(\alpha-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$.

$$\sin(\alpha-\beta)=\sin(\alpha+(-\beta))=\sin(\alpha)\cos(-\beta)+\cos(\alpha)\sin(-\beta)=\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)$$

Answers for Problem Set 6.3

Evaluate $\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}$ to show $\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}$. Use the resulting formula to show that $\tan(2\alpha) = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}$.

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)}$$

Dividing the numerator and denominator by $\cos(\alpha)\cos(\beta)$ gives

$$\tan(\alpha+\beta) = \frac{\frac{\sin(\alpha)\cos(\beta)}{\cos(\alpha)\cos(\beta)} + \frac{\cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}}{\frac{\cos(\alpha)\cos(\beta)}{\cos(\alpha)\cos(\beta)} - \frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}} = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)}.$$

It then follows that

$$\tan(2\alpha) = \tan(\alpha+\alpha) = \frac{\tan(\alpha)+\tan(\alpha)}{1-\tan(\alpha)\tan(\alpha)} = \frac{2\tan(\alpha)}{1-\tan^2(\alpha)}.$$

Show $\tan(\alpha-\beta) = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}$.

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\sin(\alpha)\cos(\beta)-\cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)}$$

Dividing the numerator and denominator by $\cos(\alpha)\cos(\beta)$ gives

$$\tan(\alpha-\beta) = \frac{\frac{\sin(\alpha)\cos(\beta)}{\cos(\alpha)\cos(\beta)} - \frac{\cos(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}}{\frac{\cos(\alpha)\cos(\beta)}{\cos(\alpha)\cos(\beta)} + \frac{\sin(\alpha)\sin(\beta)}{\cos(\alpha)\cos(\beta)}} = \frac{\tan(\alpha)-\tan(\beta)}{1+\tan(\alpha)\tan(\beta)}.$$