Problem Set

Factor each equation and solve for the solutions.

a. $x^3 + 125 = 0$

$(x+5)(x^2-5x+25)=0$

$(x+5)\left(x-\left[\frac{5+5\sqrt{3}i}{2}\right]\right)\left(x-\left[\frac{5-5\sqrt{3}i}{2}\right]\right)=0$

$x=\frac{5\pm\sqrt{25-4(1)(25)}}{2(1)}=\frac{5\pm\sqrt{25-100}}{2}$

$=\frac{5\pm\sqrt{-75}}{2}=\frac{5\pm\sqrt{75}i}{2}=\frac{5\pm5\sqrt{3}i}{2}$

$x=-5$

b. $x^3 - 27 = 0$

$(x-3)(x^2+3x+9)=0$

$(x-3)\left(x-\left[\frac{-3+3\sqrt{3}i}{2}\right]\right)\left(x-\left[\frac{-3-3\sqrt{3}i}{2}\right]\right)=0$

$x=\frac{-3\pm\sqrt{9-4(1)(9)}}{2(1)}=\frac{-3\pm\sqrt{9-36}}{2}$

$=\frac{-3\pm\sqrt{-27}}{2}=\frac{-3\pm\sqrt{27}i}{2}=\frac{-3\pm3\sqrt{3}i}{2}$

$x=3$

c. $x^4 + 9x^2 + 18 = 0$ $\quad u=x^2$

$u^2+9u+18=0$

$(u+3)(u+6)=0$

$(x^2+3)(x^2+6)=0$

$(x+\sqrt{3}i)(x-\sqrt{3}i)(x+\sqrt{6}i)(x-\sqrt{6}i)=0$

$x=\pm\sqrt{3}i \quad x=\pm\sqrt{6}i$

d. $x^4 - 7x^2 - 18 = 0$

$(x^2-9)(x^2+2)=0$

$(x-3)(x+3)(x+\sqrt{2}i)(x-\sqrt{2}i)=0$

$x=\pm3$

$x=\pm\sqrt{2}i$

e. $x^4 + 6x^2 - 40 = 0$

$(x^2+10)(x^2-4)=0$

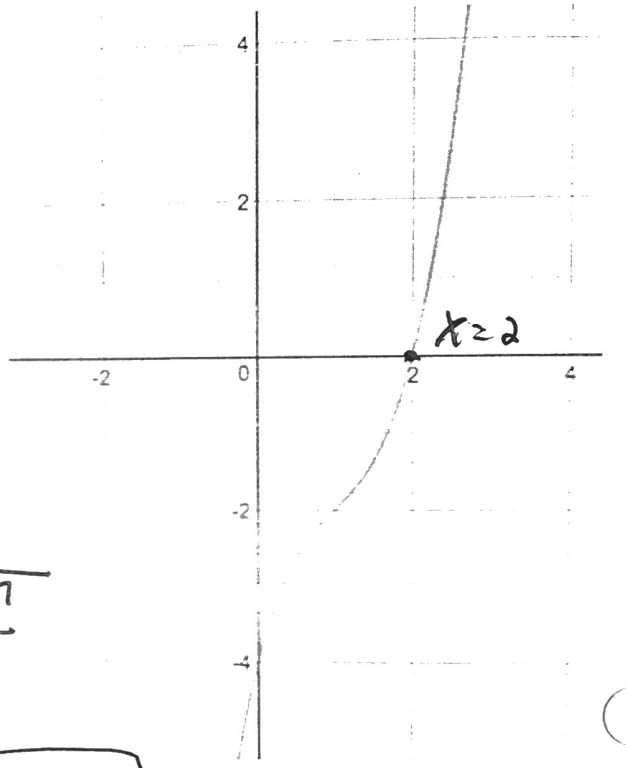
$(x+\sqrt{10}i)(x-\sqrt{10}i)(x+2)(x-2)=0$

$x=\pm\sqrt{10}i$

$x=\pm2$

6. Consider the polynomial $r(x) = x^3 - 3x^2 + 4x - 4$.

Factor r(x) completely using the real root from the graph and what you know about complex roots.



(x-2) must be a factor!
use synthetic div!

2	1	-3	4	-4
	↓	2	-2	4
	1	-1	2	0

$r(x) = (x-2)(x^2 - x + 2)$

$x = \frac{1 \pm \sqrt{1 - 4(1)(2)}}{2(1)} = \frac{1 \pm \sqrt{-7}}{2}$

$= \frac{1 \pm \sqrt{7}\,i}{2}$ → zeros

$$r(x) = (x-2)\left(x - \left[\frac{1+\sqrt{7}\,i}{2}\right]\right)\left(x - \left[\frac{1-\sqrt{7}\,i}{2}\right]\right)$$

+Khan Sets listed on my website (Due 8 AM Monday 11/27)