Problem Set

6. Find the inverse of each function below.

a. $h(x) = (3x + 7)^2$

b. $h(x) = \sqrt[3]{x^2 - 8}$

c. $h(x) = \frac{1}{2x-3}$

d. $f(x) = \frac{3x-7}{5}$

e. $f(x) = \frac{5+x}{6-2x}$

f. $f(x) = e^{x-5}$

g. $f(x) = 2^{5-8x}$

a. $y = (3x+7)^2$
$x = (3y+7)^2$
$\sqrt{x} = 3y+7$
$\sqrt{x}-7 = 3y$
$\frac{\sqrt{x}-7}{3} = y$
$f^{-1}(x) = \frac{\sqrt{x}-7}{3}$

b. $y = \sqrt[3]{x^2-8}$
$x = \sqrt[3]{y^2-8}$
$x^3 = y^2 - 8$
$x^3 + 8 = y^2$
$\sqrt{x^3+8} = y$
$f^{-1}(x) = \sqrt{x^3+8}$

c. $y = \frac{1}{2x-3}$
$x = \frac{1}{2y-3}$
$2y-3 = \frac{1}{x}$
$2y = \frac{1}{x} + 3$
$y = \frac{\frac{1}{x}+3}{2}$
$f^{-1}(x) = \frac{\frac{1}{x}+3}{2}$

d. $y = \frac{3x-7}{5}$
$x = \frac{3y-7}{5}$
$5x = 3y - 7$
$5x + 7 = 3y$
$\frac{5x+7}{3} = y$
$f^{-1}(x) = \frac{5x+7}{3}$

e. $y = \frac{5+x}{6-2x}$
$x = \frac{5+y}{6-2y}$
$(6-2y)x = 5+y$
$6x - 2xy = 5 + y$
$6x - 5 = y + 2xy$
$6x - 5 = y(1+2x)$
$\frac{6x-5}{1+2x} = y$
$f^{-1}(x) = \frac{6x-5}{1+2x}$

f. $y = e^{x-5}$
$x = e^{y-5}$
$\ln(x) = y - 5$
$\ln(x) + 5 = y$
$f^{-1}(x) = \ln(x) + 5$

g. $y = 2^{5-8x}$
$x = 2^{5-8y}$
$\log_2(x) = 5 - 8y$
$\log_2(x) - 5 = -8y$
$\frac{\log_2(x)-5}{-8} = y$
$f^{-1}(x) = \frac{\log_2(x)-5}{-8}$

7. Suppose that the inverse of a function is the function itself. For example, the inverse of the function $f(x) = \frac{1}{x}$ (for $x \neq 0$) is just itself again, $g(x) = \frac{1}{x}$ (for $x \neq 0$). What symmetry must the graphs of all such functions have? (Hint: Study the graph of Exercise 5 in the lesson.)

All of these kinds of graphs must have reflective symmetry over $y = x$ b/c that is how inverses are formed graphically.