Lesson Summary

Pascal's triangle is an arrangement of numbers generated recursively:

Row 0:						1					
Row 1:					1		1				
Row 2:				1		2		1			
Row 3:			1		3		3		1		
Row 4:		1		4		6		4		1	
Row 5:	1		5		10		10		5		1
	⋮		⋮		⋮		⋮		⋮		⋮

For an integer $n \geq 1$, the number $n!$ is the product of all positive integers less than or equal to n.
We define $0! = 1$.

The binomial coefficients $C(n, k)$ are given by $C(n, k) = \frac{n!}{k!(n-k)}!$ for integers $n \geq 0$ and $0 \leq k \leq n$.

THE BINOMIAL THEOREM: For any expressions u and v,

$(u + v)^n = u^n + C(n, 1)u^{n-1}v + C(n, 2)u^{n-2}v^2 + \cdots + C(n, k)u^{n-k}v^k + \cdots + C(n, n-1)u\, v^{n-1} + v^n.$

That is, the coefficients of the expanded binomial $(u + v)^n$ are exactly the numbers in Row n of Pascal's triangle.

Problem Set

Use the binomial theorem to expand the following binomial expressions.

a. $(x + y)^4$
b. $(x + 2y)^4$
c. $(x + 2xy)^4$
d. $(x - y)^4$
e. $(x - 2xy)^4$

a. $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b. $x^4 + 4x^3(2y) + 6x^2(2y)^2 + 4x(2y)^3 + (2y)^4$
$x^4 + 8x^3y + 6x^2 4y^2 + 4x8y^3 + 16y^4$
$x^4 + 8x^3y + 24x^2y^2 + 32xy^3 + 16y^4$

c. $x^4 + 4x^3(2xy) + 6x^2(2xy)^2 + 4x(2xy)^3 + (2xy)^4$
$x^4 + 8x^3xy + 24x^2x^2y^2 + 32xx^3y^3 + 16x^4y^4$
$x^4 + 8x^4y + 24x^4y^2 + 32x^4y^3 + 16x^4y^4$

d. $x^4 + 4x^3(-y) + 6x^2(-y)^2 + 4x(-y)^3 + (-y)^4$
$x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

e. $x^4 - 8x^4y + 24x^4y^2 - 32x^4y^3 + 16x^4y^4$

Use the binomial theorem to expand the following binomial expressions.

a. $(1+\sqrt{2})^5$ $= 1+5\sqrt{2}+10\cdot 2+10\cdot 2\sqrt{2}+5\cdot 4+4\sqrt{2} = 41+29\sqrt{2}$

b. $(1+i)^9$ $= 1+9i+36i^2+84i^3+126i^4+126i^5+84i^6+36i^7+9i^8+i^9$

$= 1+9i-36-84i+126-126i-84-36i+9+i = 16+16i$

Consider the expansion of $(x+y)^{10}$. Determine the coefficients for the terms with the powers of x and y shown.

a. x^2y^8 $C(10,8)=45$ $n=10$ $y^8 \to r=8$

b. x^4y^6 $C(10,6)=210$ $n=10$ $r=6$

c. x^5y^5 $C(10,5)=252$ $n=10$ $r=5$

Consider the expansion of $(5p+2q)^6$. Determine the coefficients for the terms with the powers of p and q shown.

a. p^2q^4 $\Rightarrow (5p)^2(2q)^4 \Rightarrow 5^2\cdot 2^4\cdot C(6,4) = 25\cdot 16\cdot 15 = 6000$

b. p^5q $(5p)^5(2q) \Rightarrow 5^5\cdot 2\cdot C(6,1) = 3125(2)6 = 37500$

c. p^3q^3 $(5p)^3(2q)^3 = 5^3\cdot 2^3\,(6,3) = 125\cdot 8\cdot 20 = 20000$