Lesson 4: Solutions to Quadratic Equations

Classwork

Opening Exercise

How many solutions are there to the equation $x^2 = 1$? Explain how you know.

Two, $x=1$ and $x=-1$

$1^2=1$

$(-1)^2=1$

Prove that 1 and -1 are the only solutions to the equation $x^2 = 1$.

How can we do this another way?

$x=\pm\sqrt{1}$

$x=\pm 1$

Complex numbers:

Exercises

Find the product.

1. $(z-2)(z+2)$

z^2-4

2. $(z+3i)(z-3i)$

z^2+9

Factor Rules:

$a^2 - b^2$	$a^2 + b^2$
$(a-b)(a+b)$	$(a-bi)(a+bi)$

Write each of the following quadratic expressions as the product of two linear factors.

$z^2 - 4$ $(z-2)(z+2)$	$z^2 - 8$ $(z-\sqrt{8})(z+\sqrt{8})$
$z^2 + 4$ $(z-2i)(z+2i)$	$z^2 + 8$ $(z+\sqrt{8}i)(z-\sqrt{8}i)$
$b^2 - 9$ $(b-3)(b+3)$	$b^2 - 1$ $(b+1)(b-1)$
$v^2 + 100$ $(v+10i)(v-10i)$	$r^2 + 12$ $(r+\sqrt{12}i)(r-\sqrt{12}i)$
$x^2 + 64$ $(x+8i)(x-8i)$	$x^2 - 4i$ $(x-2\sqrt{i})(x+2\sqrt{i})$
$y^2 - 49$ $(y-7)(y+7)$	$x^2 + 4i$ $(x+2i\sqrt{i})(x-2\sqrt{i}i)$

Do ALL quadratics have two factors?

Yes

Do complex quadratics have two factors?

Yes

Stop

3. Can a quadratic polynomial equation with real coefficients have one real solution and one complex solution? If so, give an example of such an equation. If not, explain why not.

Recall from Algebra II that every quadratic expression can be written as a product of two linear factors, that is,

$$ax^2 + bx + c = a(x - r_1)(x - r_2),$$

where r_1 and r_2 are solutions of the polynomial equation $ax^2 + bx + c = 0$.

4. Solve each equation by factoring, and state the solutions.

   a. $x^2 + 25 = 0$

   $(x+5i)(x-5i) = 0$

   $x = \pm 5i$

   b. $x^2 + 10x + 25 = 0$

   $(x+5)(x+5) = 0$

   $x = -5$

c. $x^2 + 81 = 0$

$(x+9i)(x-9i)=0$

$x = \pm 9i$

d. $x^2 + 8x + 12 = 0$

$(x+6)(x+2)=0$

$x=-6, -2$

e. $x^2 - 144 = 0$

$(x-12)(x+12)=0$

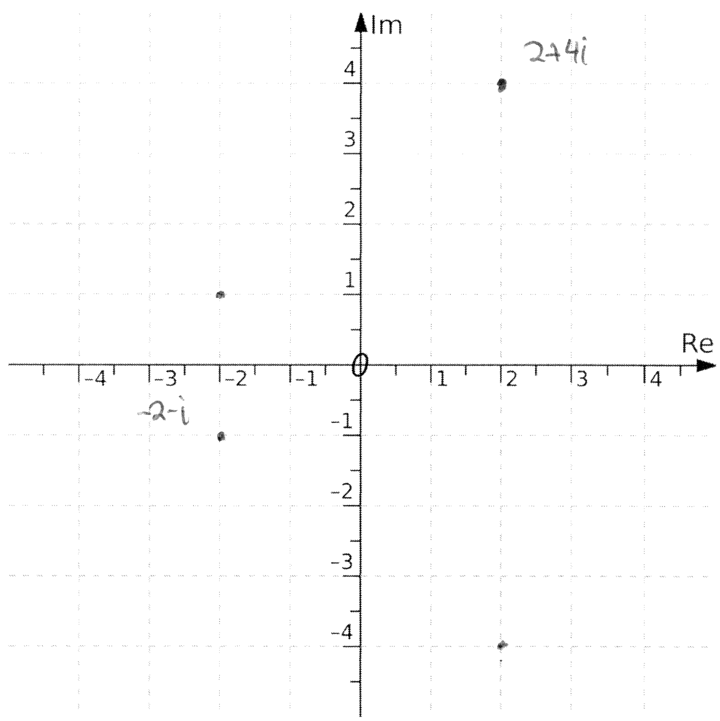
$x=-12, 12$

Complex conjugates: In linear factors, complex solutions always come in conjugate pair.

If a+bi is a solution, then a - bi is also a solution.

Graph $2 + 4i$ and its conjugate.

Graph $-2 - i$ and its conjugate.



What do you observe about a number and its conjugate?

Reflected over Real axis

What do you observe about the modulus of a number and its conjugate?

Modulus same

Example: Let's say that a quadratic with real coefficients has one known factor of $(x-(5-3i))$. What is the other factor?

$(x - (5+3i))$

What are the solutions of that quadratic?

$y = (x - (5+3i))(x-(5-3i)) = x^2 \ldots$

$(5+3i)$ $(5-3i)$

Examples: Give an example of a quadratic equation with $2+3i$ as one of its solutions.

$(x-(2+3i))(x-(2-3i))$

Practice 1: Let's say that a quadratic with real coefficients has one known factor of $(x-(9+2i))$. What is the other factor?

$(x-(9+2i))(x-(9-2i))$

What are the solutions of that quadratic?

$(9+2i), (9-2i)$

Practice 2: Let's say that a quadratic with real coefficients has one known factor of $(x+(-2+i))$. What is the other factor?

What are the solutions of that quadratic?

$(-2+i), (-2-i)$

Fundamental Theorem of Algebra

1. Every polynomial function of degree $n \geq 1$ with real or complex coefficients has at least _______real or complex zero.

2. Every polynomial of degree $n \geq 1$ with real or complex coefficients can be factored into ________linear terms with real or complex coefficients.

5. Write the left side of each equation as a product of linear factors, and state the solutions.

   a. $x^3 - 1 = 0$ $a^3 - b^3 =$

   $(x-1)(x^2+x+1)$

   b. $x^3 + 8 = 0$ $a^3 + b^3 =$

   $(x+2)(x^2-2x+4)$

Exercises

6. Write the left side of each equation as a product of linear factors, and state the solutions.

   a. $x^3 + 27 = 0$

   b. $x^3 - 8 = 0$

Homework

Find all solutions to the following quadratic equations, and write each equation in factored form.

c. $x^2 + 25 = 0$

$(x+5i)(x-5i)=0$

$x=\pm 5i$

d. $x^2 - 16 = -7$

$x^2-9=0$

$(x+3)(x-3)=0$

$x=\pm 3$

e. $x^2 + 11x + 28 = 0$

$(x+7)(x+4)=0$

$x=-7$

$x=-4$

Practice 1: Let's say that a quadratic with real coefficients has one known factor of $(x - (11 - 7i))$. What is the other factor?

$(x-(11+7i))$ is the other factor

What are the solutions of that quadratic?

$x=11+7i$

$x=11-7i$

Practice 2: Let's say that a quadratic with real coefficients has one known factor of $(x + (-4 - 2i))$. What is the other factor?

$(x+(-4+2i))$ is the other factor.

What are the solutions of that quadratic?

$x=4-2i$

$x=4+2i$