

Lesson 3: Arithmetic with Complex Numbers

Practice Multiplying complex numbers.

Do Now: Find the product of $(3+i)$ and $(2+i)$.

$$\begin{aligned} &(3+i)(2+i) \\ &6+3i+2i+i^2 \\ &\boxed{5+5i} \end{aligned}$$

Find the product of $(-3+2i)$ and $(5+3i)$.

$$\begin{aligned} &(-3+2i)(5+3i) \\ &-15-9i+10i+6i^2 \\ &-15+i-6 \\ &\boxed{-21+i} \end{aligned}$$

Find the sum of $2+3i$ and $-5-7i$.

$$\begin{aligned} &(2+3i)+(-5-7i) \\ &2+3i-5-7i \\ &\boxed{-3-4i} \end{aligned}$$

Practice:

Simplify.

1) $i+6i$

$7i$

2) $3+4+6i$

$7+6i$

3) $3i+i$

$4i$

4) $-8i-7i$

$-15i$

5) $-1-8i-4-i$

$-5-9i$

6) $7+i+4+4$

$15+i$

7) $-3 + 6i - (-5 - 3i) - 8i$

$2 + i$

8) $3 + 3i + 8 - 2i - 7$

$4 + i$

9) $4i(-2 - 8i)$

$32 - 8i$

10) $5i \cdot -i$

5

13) $(-2 - i)(4 + i)$

$-7 - 6i$

14) $(7 - 6i)(-8 + 3i)$

$-38 + 69i$

15) $7i \cdot 3i(-8 - 6i)$

$168 + 126i$

16) $(4 - 5i)(4 - i)$

$21 - 16i$

17. $(x + 3i)(x - 3i)$

$x^2 + 9$

18. $(x + 3i)(x - i)(x + i)(x - 3i)$

$(x^2 + 9)(x^2 + 1)$

$x^4 + 10x^2 + 9$

19. $(x + i)^2 \cdot (x - i)^2$

$(x + i)(x - i)(x + i)(x - i)$

$(x^2 + 1)(x^2 + 1)$

$x^4 + 2x^2 + 1$

Patterns in complex numbers:

Remember, $i = \sqrt{-1}$ So... $i^0 = \underline{1}$

$i^2 = \underline{-1}$

$i^3 = \underline{-i}$

$i^4 = \underline{1}$

$i^5 = \underline{i}$

What is i^{18} ?

-1

What is i^{23} ?

$-i$

What is i^{17} ?

i

What is i^{41} ?

i

Find the power of i .

23) i^{16}

A) 1

B) $-i$

C) -1

D) i

24) i^{19}

A) $-i$

B) -1

C) i

D) 1

25) i^{21}

A) i

B) -1

C) 1

D) $-i$

26) i^{14}

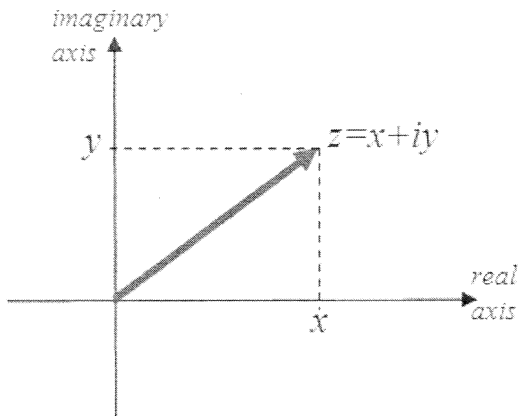
A) i

B) -1

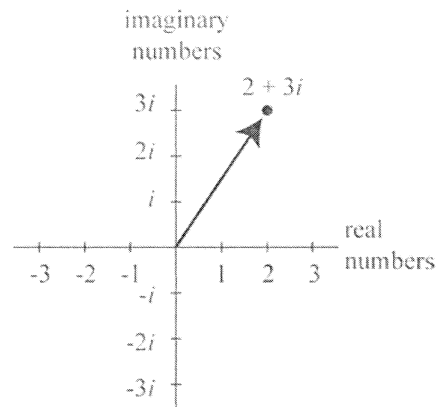
C) 1

D) $-i$

Graphing Complex Numbers

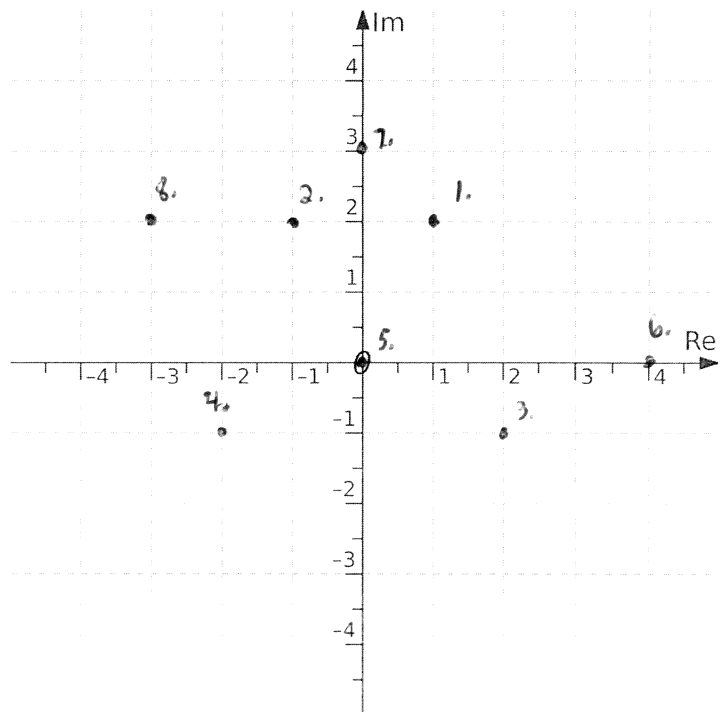


Graph the number $2+3i$



Graph the following numbers on the axis at the right. Label them after you graph them. You do not need to draw an arrow to the point.

1. $1 + 2i$
2. $-1 + 2i$
3. $2 - i$
4. $-2 - i$
5. 0
6. 4
7. $3i$
8. $-3 + 2i$



MODULUS

Graph the point $3 + 4i$. Label that point P. Draw a line from O to P. Turn this into a triangle with base along the REAL AXIS. The distance OP is called the modulus of $3 + 4i$.

Find the modulus OR $|3 + 4i|$.

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

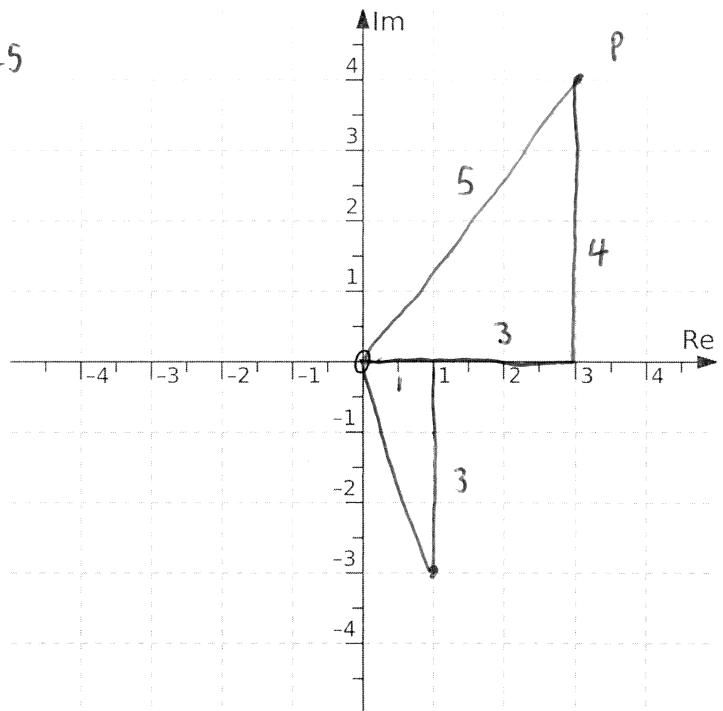
Follow the same steps with the number $1 - 3i$.

Find $|1 - 3i|$.

$$1^2 + 3^2 = c^2$$

$$\sqrt{1 + 9} = c$$

$$\boxed{\sqrt{10} = c}$$



Find the absolute value of each complex number.

1) $|7 - i|$

$5\sqrt{2}$

2) $|-5 - 5i|$

$5\sqrt{2}$

3) $|-2 + 4i|$

$2\sqrt{5}$

4) $|3 - 6i|$

$3\sqrt{5}$

5) $|10 - 2i|$

$2\sqrt{26}$

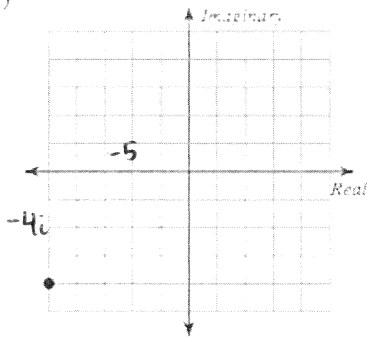
6) $|-4 - 8i|$

$4\sqrt{5}$

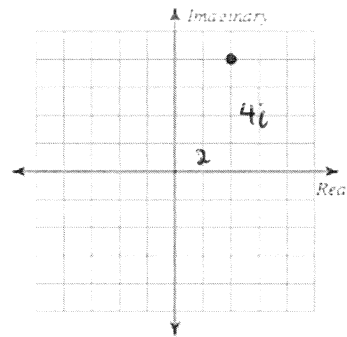
HOMEWORK

Identify each complex number graphed.

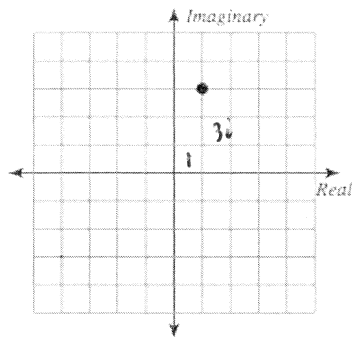
17)



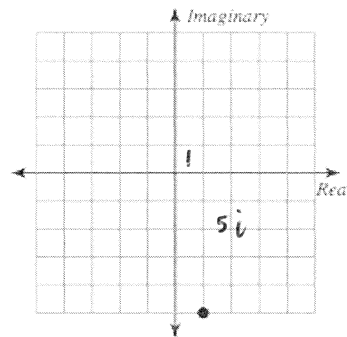
18)



19)

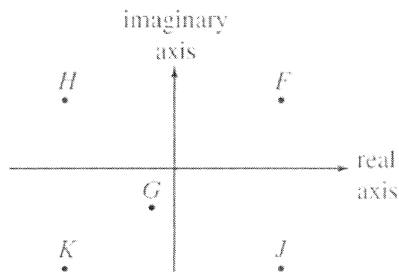


20)



Which point has the smallest modulus?

48. In a complex plane, the vertical axis is the *imaginary axis* and the horizontal axis is the *real axis*. Within the complex plane, a complex number $a + bi$ is comparable to the point (a, b) in the standard (x, y) coordinate plane. $\sqrt{a^2 + b^2}$ is the modulus of the complex point $a + bi$. Which of the complex numbers $F, G, H, J,$ and K below has the smallest modulus?



↳ distance from origin

- F. F
- G. G
- H. H
- J. J
- K. K

Subtract a from b , given:

$$a = 3 + i$$

$$b = 4 - 2i$$

Possible Answers:

$$-1 + 3i$$

$$1 - 3i$$

$$3 + i$$

$$1 + 3i$$

$$-1 - 3i$$

$$(4 - 2i) - (3 + i)$$

$$4 - 2i - 3 - i$$

$$1 - 3i$$

Complex numbers take the form $a + bi$, where a is the real term in the complex number and bi is the nonreal (imaginary) term in the complex number.

Which of the following equations simplifies into $4 + 2i$?

Possible Answers:

$$(8 - 5i) + (-4 + 7i) = 4 + 2i$$

$$(10 - i) + (6 - i) = 4 + 0i$$

$$(0 + 7i) + (-4 - 5i) = 4 + 2i$$

$$3 + (7 + 2i) = -4 - 2i$$

$$0 + 6i + (-4 - 4i) = -4 + 2i$$