Lesson Summary

- Polynomial equations with real coefficients can have real or complex solutions or they can have both.
- Complex solutions to polynomial equations with real coefficients are always members of a conjugate pair.
- Real solutions to polynomial equations are the same as x-intercepts of the associated graph, but complex solutions are not.

Problem Set

1. Rewrite each expression in standard form (distribute out the factors).

$(x + 3i)(x - 3i)$

$\boxed{x^2+9}$

$(x + 2i)(x - i)(x + i)(x - 2i)$

$(x+2i)(x-2i)(x-i)(x+i)$

$(x^2+4)(x^2+1)$

$x^4+x^2+4x^2+4$

$\boxed{x^4+5x^2+4}$

$(x + i)^2 \cdot (x - i)^2$

$(x+i)(x-i)(x+i)(x-i)$

$(x^2+1)\ (x^2+1)$

$x^4+x^2+x^2+1$

$\boxed{x^4+2x^2+1}$

Write a polynomial equation of degree 4 in standard form that has the solutions $i, -i, 1, -1$.

$y=(x-i)(x+i)(x-1)(x+1)$

$y=(x^2+1)(x^2-1)$

$\boxed{y=x^4-1}$

Explain the difference between x-intercepts and solutions to an equation. Give an example of a polynomial with real coefficients that has twice as many solutions as x-intercepts. Write it in standard form.

Solutions are x-values that make an equation true. If a polynomial expression is equal to zero, those solutions are the x-intercepts of the corresponding function if the roots are real.

$y = (x-1)(x+i)(x-i)(x-2)$ 4 solutions, 2 x-int

$y = (x^2-3x+2)(x^2+1)$

$y = x^4 + x^2 - 3x^3 - 3x + 2x^2 + 2$

$y = x^4 - 3x^3 + 3x^2 - 3x + 2$

Find the solutions to $x^4 - 5x^2 - 36 = 0$ and the x-intercepts of the graph of $y = x^4 - 5x^2 - 36$.

$(x^2-9)(x^2+4)=0$

$(x-3)(x+3)(x+2i)(x-2i)=0$

$\boxed{\begin{array}{c} x = \pm 3 \\ x = \pm 2i \end{array}}$

treat $x^4 - 5x^2 - 36 = 0$ as $y^2 - 5y - 36 = 0$ where $y = x^2$

$(y-9)(y+4)=0$

$(x^2-9)(x^2+4)=0$