Lesson 6.2: Introduction to Radians and the Unit Circle
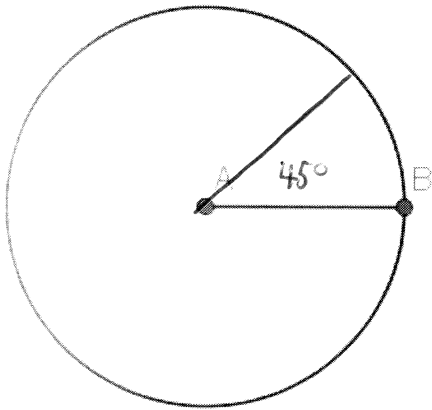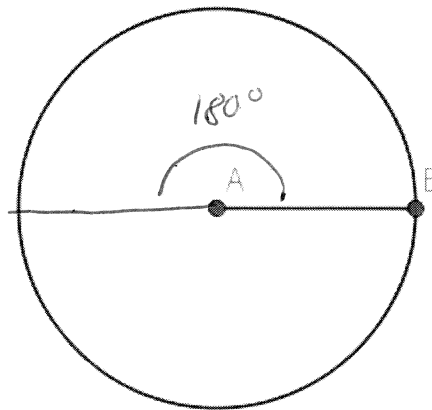
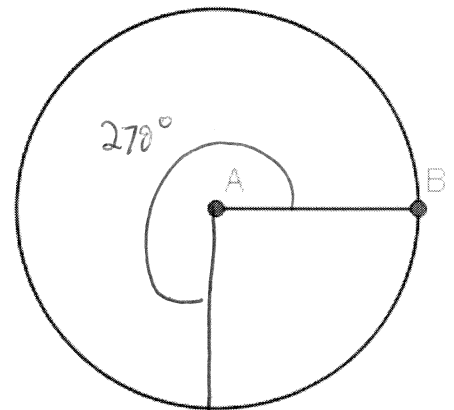Opening **Exercise**: Sketch the given angle.
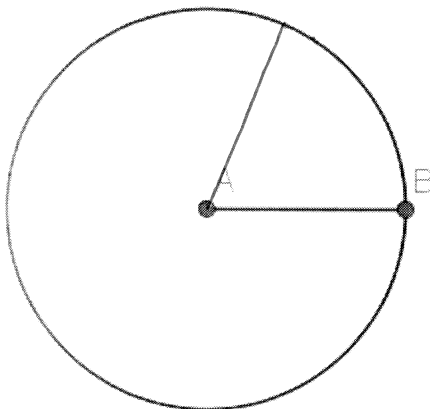
45° — 180° — 270°

What we just sketched were angles in ___degrees___.

But what happens when the angle gets even bigger?

There are ___360°___ degrees in ONE full rotation. There are ___720°___ degrees in TWO full rotations.

What's the problem? As we create more rotations, the angle value increases QUICKLY. Let's make a NEW system.

Let's call the radius, r. Label the circle with me.

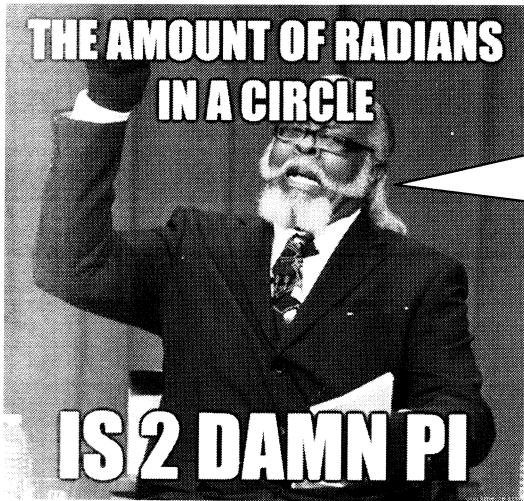The ___angle___ were the ___length of arc___ and ___radius___ are equal is called ___radians___.

There are ___π___ radians in half a circle.

That means there are ___π___ radians for every 180°.

How many radians in 360°?

___2π___

A reminder

$$\frac{180°}{360°} = \frac{\pi}{2\pi}$$

Proportions solve problems!

$$\text{radians}\left(\frac{180}{\pi}\right)$$

Examples:

$$\text{degrees}\left(\frac{\pi}{180}\right)$$

Convert the following radian measures to degrees.

$$\frac{\pi}{3} \cdot \frac{180}{\pi} = 60°$$

$$\frac{7\pi}{6} \cdot \frac{180}{\pi} = 210°$$

$$4\pi\left(\frac{180}{\pi}\right) = 720°$$

Convert the following degree measures to radians.

$$135°\left(\frac{\pi}{180}\right) = \frac{3\pi}{4}$$

$$270°\left(\frac{\pi}{180}\right) = \frac{3\pi}{2}$$

$$900°\left(\frac{\pi}{180}\right) = 5\pi$$

Convert the given measure to the OTHER system of angle measure.

1) $-290° = \dfrac{-29\pi}{18}$

2) $345° = \dfrac{23\pi}{12}$

3) $970° = \dfrac{97\pi}{18}$

4) $-510° = \dfrac{-17\pi}{6}$

5) $510° = \dfrac{17\pi}{6}$

6) $150° = \dfrac{5\pi}{6}$

21) $\dfrac{\pi}{18} = 10°$

22) $-\dfrac{25\pi}{12} = -375°$

23) $\dfrac{35\pi}{18} = 350°$

24) $\dfrac{41\pi}{36} = 205°$

25) $-\dfrac{3\pi}{2} = -270°$

26) $\dfrac{107\pi}{36} = 535°$

Discussion
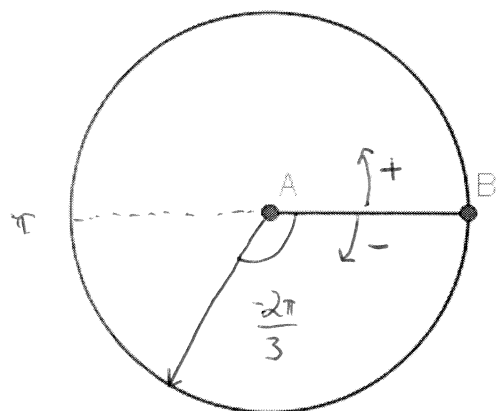
Positive angles in standard position rotate:

Counterclockwise

Negative angles in standard position rotate:

Clockwise

Sketch an angle of $-\dfrac{2\pi}{3}$ on the circle given.

$\sin(60) = \dfrac{\sqrt{3}}{2}$

$\cos(60) = \dfrac{1}{2}$

$\tan(60) = \sqrt{3}$

$\sin(45) = \dfrac{\sqrt{2}}{2}$
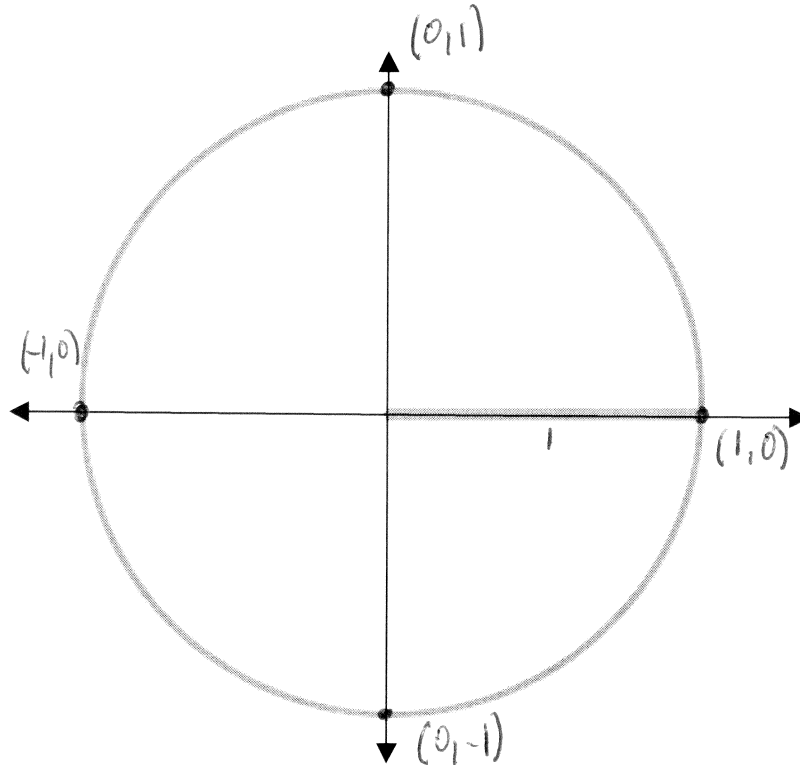
$\cos(45) = \dfrac{\sqrt{2}}{2}$
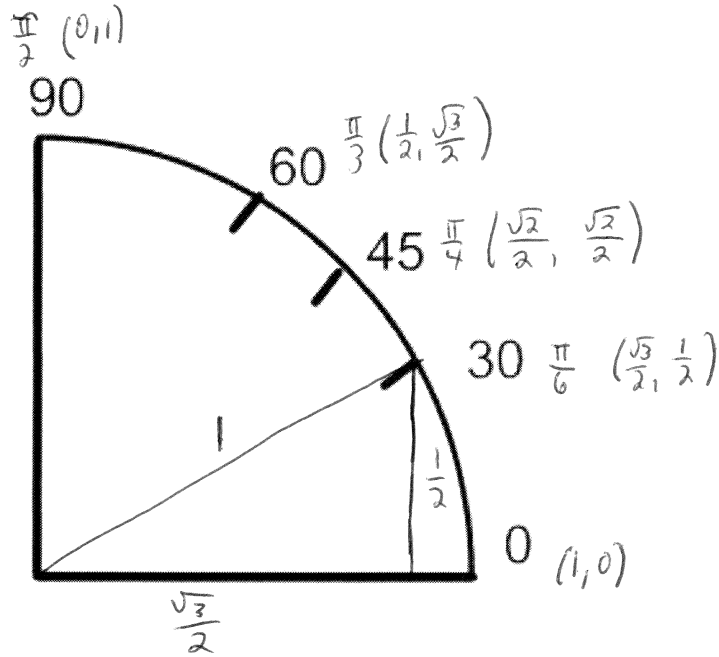
$\tan(45) = 1$

$\sin(30) = \dfrac{1}{2}$

$\cos(30) = \dfrac{\sqrt{3}}{2}$

$\tan(30) = \dfrac{\sqrt{3}}{3}$

The Unit Circle

One of our best tools in trigonometry is called the unit circle. It is a UNIT CIRCLE because the radius of the

circle is _____One unit_____. Label this circle with me.

$(0,1)$

$(-1,0)$

$(1,0)$

$(0,-1)$

$\frac{\pi}{2}$ $(0,1)$
90

60 $\frac{\pi}{3}$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

45 $\frac{\pi}{4}$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

30 $\frac{\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{1}{2}$

0 $(1,0)$

$\frac{\sqrt{3}}{2}$

SO: In the unit circle, the coordinates of the points are (_____cosine_____,_____sine_____)

MNEMONIC: (x, y) (c, s)

Example: $\frac{\pi}{6}$ has a coordinate of $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

What is the cosine of $\frac{\pi}{3}$?

$\frac{1}{2}$
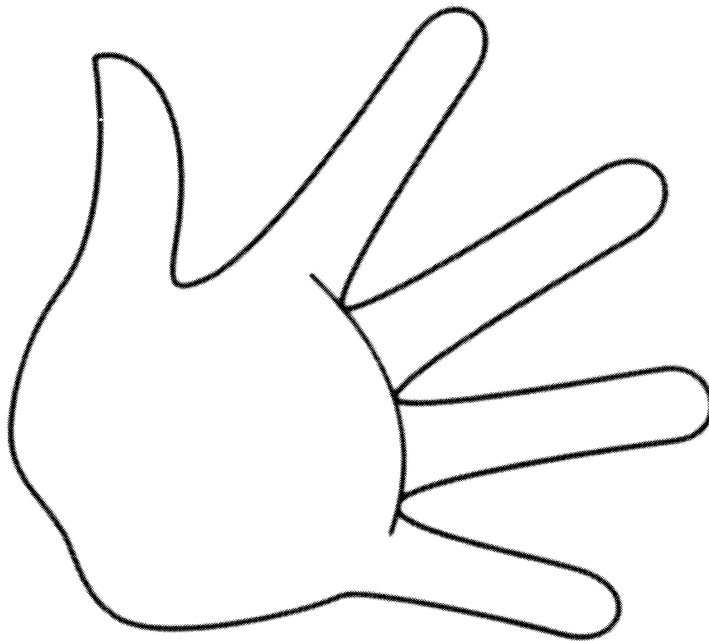
What is the sine of $\frac{\pi}{2}$?

1

What is the tangent of $\frac{\pi}{4}$?

1

Two Ways to Remember

Table:

Left Hand:

Use the Left Hand Trick to evaluate the following expressions.

$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\tan(0) = 0$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$

$\tan\left(\frac{\pi}{3}\right) = \sqrt{3}$

$\sec\left(\frac{\pi}{3}\right) = 2$

$\sin(0) = 0$

$\cos(0) = 1$

$\tan\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$
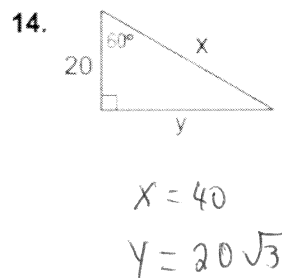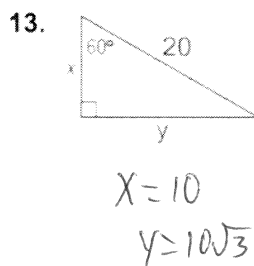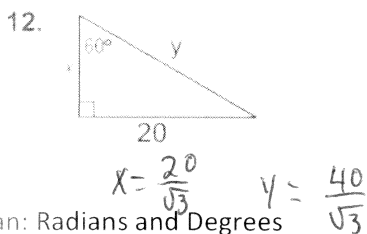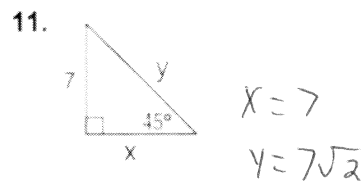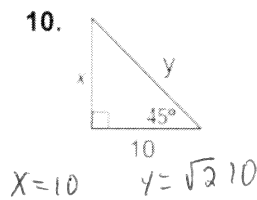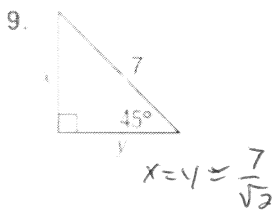
$\sin\left(\frac{\pi}{2}\right) = 1$

$\cot\left(\frac{\pi}{6}\right) = 3$

$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

$\cot\left(\frac{\pi}{4}\right) = 1$

$\sec\left(\frac{\pi}{2}\right) = undef$

Find the value of x and y in each figure.

9.

$x = y = \frac{7}{\sqrt{2}}$

10.

$X = 10 \quad y = \sqrt{2}\,10$

11.

$X = 7$

$Y = 7\sqrt{2}$

12.

$X = \frac{20}{\sqrt{3}} \quad Y = \frac{40}{\sqrt{3}}$

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13.

$X = 10$

$Y = 10\sqrt{3}$

14.

$X = 40$

$Y = 20\sqrt{3}$