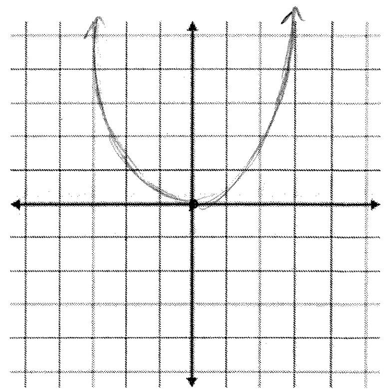
4.4 f(x) and f'(x)

DO NOW: Sketch a smooth curve y=f(x) through the origin with the properties that f'(x)<0 for x<0 and f'(x)>0 for x>0.



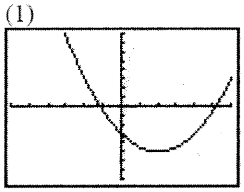
f(x<0) - decreasing

f(x>0) = increasing

Match the graph of f(x) to the graph of f'(x)

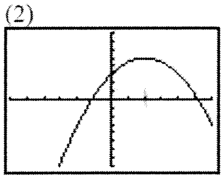
FUNCTIONS

(1)



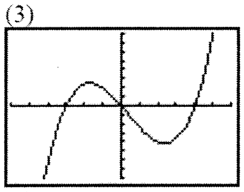
E

(2)



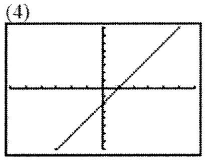
B

(3)



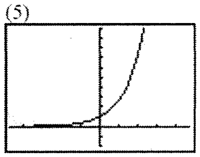
G

(4)



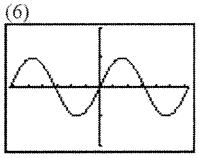
A

(5)



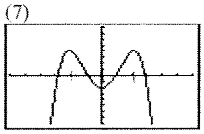
F

(6)



D

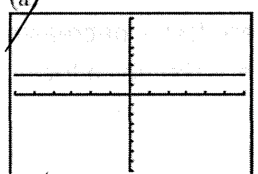
(7)



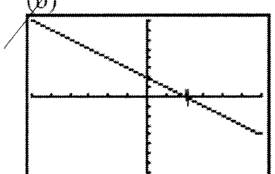
C

DERIVATIVES

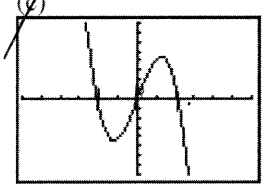
(a)



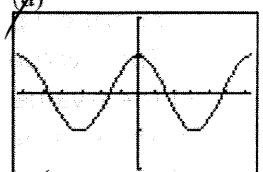
(b)



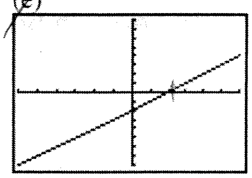
(c)



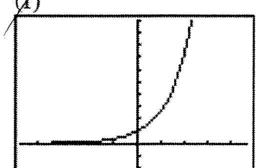
(d)



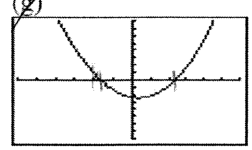
(e)

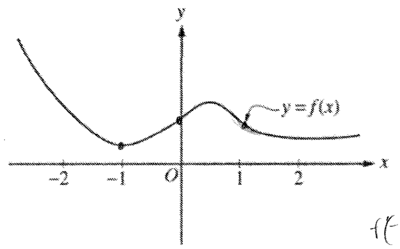


(f)



(g)





f(-1)=0   f'(1)= -   f'(0)=+

88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

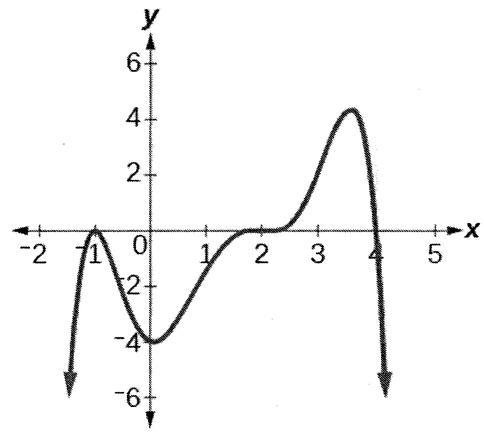
(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$

Group 1: Using the graph of f'(x) given at right, identify
(a) all critical points of f(x),
(b) The x-values where f(x) is increasing
(c) The x-values where f(x) is decreasing
(d) The x-values where f(x) has a local maximum,
(e) The x-values where f(x) has a local minimum
and explain how you know.



a) when f'(x) = 0 or DNE
CP: x = -1, x = 2, x = 4

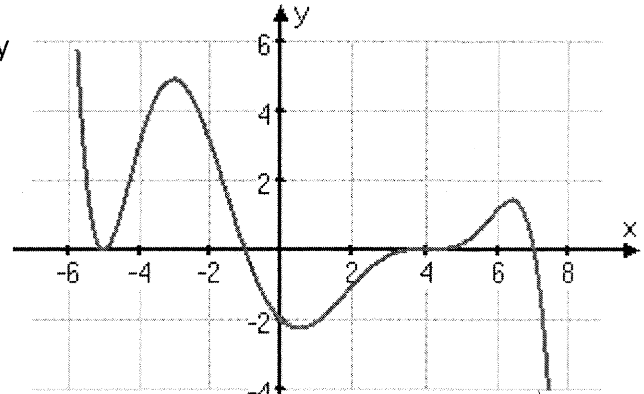
b) when f'(x) is positive, ∴
(2, 4)

c) when f'(x) is negative, ∴
(-∞, -1) (-1, 2) (4, ∞)

d) f(x) has local max when f'(x) changes fr. + → -
∴ @ x = 4

e) f(x) has local min when f'(x) changes fr. - → +
∴ @ x = 2

Group 2: Using the graph of f'(x) given at right, identify
(a) all critical points of f(x),
(b) The x-values where f(x) is increasing
(c) The x-values where f(x) is decreasing
(d) The x-values where f(x) has a local maximum,
(e) The x-values where f(x) has a local minimum
and explain how you know.



a) when $f'(x) = 0$ or DNE
CP: $x = -5$, $x = -1$, $x = 4$, $x = 7$
b) f(x) is increasing when f'(x) is pos, ∴
$(-\infty, -5)(-5, -1)(4, 7)$
c) f(x) is decreasing when f'(x) is neg ∴
$(-1, 4)(7, \infty)$
d) f(x) has local max when f'(x) changes fr. + → −
@ $x = -1$, $x = 7$

e) f(x) has local min when f'(x) changes fr. − → + ∴ @ $x = 4$

-1.75 $-1\frac{3}{4}$ $-\frac{7}{4}\left(\frac{2}{1}\right) - -\frac{7}{2} + 3$

Practice: Use the First Derivative Test to identify any local extremes. Then identify the intervals of increasing and decreasing.
$f(x)=4x^3+21x^2+36x-20$

$f'(x) = 12x^2 + 42x + 36$
$6(2x^2 + 7x + 6)$
$6(2x+3)(x+2)$
$mn = 6$
$2n+m = 7$
2 3

	↗	-2 ↘	$-\frac{3}{2}$ ↗
$2x+3$	−	−	+
$x+2$	−	+	+
	+	−	+

$\boxed{\text{Inc}}$ $(-\infty, -2)(-\frac{3}{2}, \infty)$
$\boxed{\text{Dec}}$ $(-2, -\frac{3}{2})$
$\boxed{\text{Rel max}}$ @ $x = -2$
$\boxed{\text{Rel min}}$ @ $x = -\frac{3}{2}$

$f(x)=12x^2-24x$

$f'(x) = 24x - 24$
$24(x-1)$

CP: $x = 1$

	↘	1 ↗
$x-1$	−	+

$\boxed{\text{Dec}}$ $(-\infty, 1)$
$\boxed{\text{Inc}}$: $(1, \infty)$
$\boxed{\text{Rel max}}$: none
$\boxed{\text{Rel min}}$: $x = 1$

$j(x) = (x^2 + 1)e^x$

$j'(x) = (x^2+1)e^x + (2x)e^x$

$e^x(x^2+2x+1)$

$e^x(x+1)(x+1)$

CP: $x = -1$

$\nearrow$ -1 $\nearrow$

$(x+1)^2$	+	+

$\boxed{\text{Inc}}$: $(-\infty, -1)(-1, \infty)$

$\boxed{\text{Dec}}$: none

$\boxed{\text{NO REL MAX/MIN}}$

$g(x) = (x^2 - 8)e^x$

$g'(x) = e^x(x^2-8) + e^x(2x)$

$e^x(x^2+2x-8)$

$e^x(x+4)(x-2)$

CP: $x = -4$, $x = 2$

2, 4, 2, −8

$\nearrow$ -4 $\searrow$ 2 $\nearrow$

$x+4$	−	+	+
$x-2$	−	−	+
	⊕	⊖	⊕

$\boxed{\text{Rel min:}}$ 2

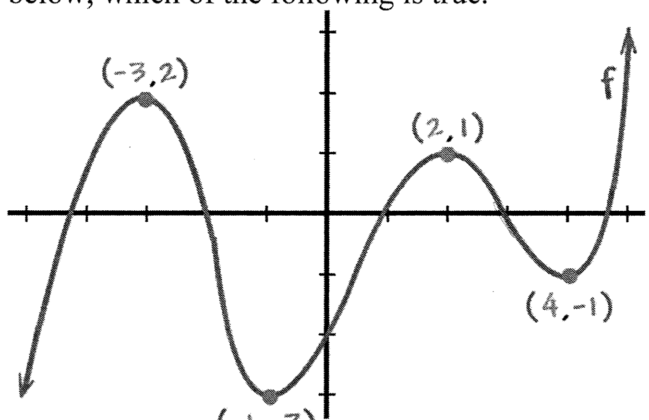
$\boxed{\text{Rel max:}}$ −4

$\boxed{\text{Inc}}$ $(-\infty, -4)(2, \infty)$

$\boxed{\text{Dec}}$ $(-4, 2)$

MC Practice:

Given the graph of f(x) below, which of the following is true.



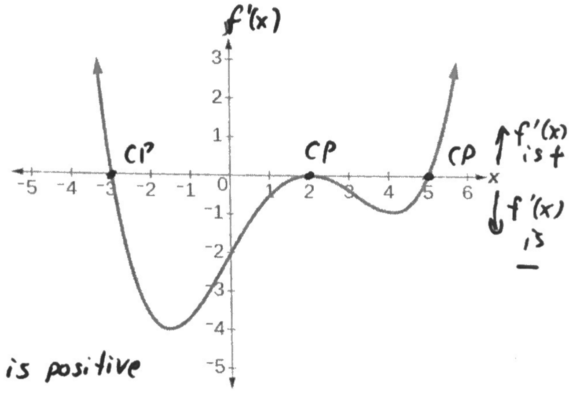
$f'(-3) = 0$

$f'(0) = +$

$f'(3) = -$

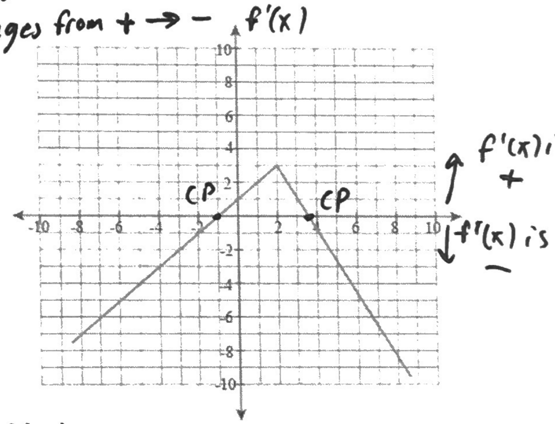
a. f'(-3)<f'(0)<f'(3)
b. f'(3)<f'(0)<f'(3)
c. f'(0)<f'(-3)<f'(3)
d. f'(3)<f'(-3)<f'(0)

Homework



HWK 1: Using the graph of f'(x) given at right, identify
(a) all critical points of f(x),
(b) The x-values where f(x) is increasing
(c) The x-values where f(x) is decreasing
(d) The x-values where f(x) has a local maximum,
(e) The x-values where f(x) has a local minimum
and explain how you know.

a) CP at $x=-3, 2, 5$ b/c $f'(x)=0$ or DNE
b) f(x) is increasing on $(-\infty,-3)$ and $(5,\infty)$ b/c f'(x) is positive
c) f(x) is decreasing on $(-3,2)$ and $(2,5)$ b/c f'(x) is negative
oops! e ~~d~~) f(x) has a local min at $x=5$ b/c f'(x) changes from $- \to +$
d) f(x) has a local max at $x=-3$ b/c f'(x) changes from $+ \to -$



HWK 2: Using the graph of f'(x) given at right, identify
(a) all critical points of f(x),
(b) The x-values where f(x) is increasing
(c) The x-values where f(x) is decreasing
(d) The x-values where f(x) has a local maximum,
(e) The x-values where f(x) has a local minimum
and explain how you know.

a) CP at $x=-1, 3.5$ b/c $f'(x)=0$ or DNE
b) f(x) is increasing on $(-1, 3.5)$ b/c f'(x) is +.
c) f(x) is decreasing on $(-\infty,-1)$ and $(3.5,\infty)$ b/c f'(x) is −
d). f(x) has local max at $x=3.5$ b/c f'(x) changes $+ \to -$
e) f(x) has a local min at $x=-1$ b/c f'(x) changes $- \to +$

Use the First Derivative Test to identify any local extremes. Then identify the intervals of increasing and decreasing.

$f(x)=x^3e^x$ $f'(x)=x^3e^x+3x^2e^x$
$f'(x)=e^x(x^3+3x^2)$
$f'(x)=e^x \cdot x^2(x+3)$

CP: $x=-3$ $x=0$

	$-$ (−3)	$+$ (0)	$+$
e^x	+	+	+
x^2	+	+	+
$(x+3)$	−	+	+
	↓	↗	↗

Dec: $(-\infty,-3)$
Inc: $(-3,0)(0,\infty)$
Min: at $x=-3$
No max

$g(x)=\frac{4x-3}{2x+1}$ $g'(x)=\frac{(2x+1)(4)-(4x-3)(2)}{(2x+1)^2}$

$g'(x)=\frac{8x+4-8x+6}{(2x+1)^2}=\frac{10}{(2x+1)^2}$

CP. $x=-\frac{1}{2}$ ← $2x+1=0$

	+ (−1/2)	+
10	+	+
$(2x+1)^2$	+	+

Dec: no where
Inc: $(-\infty,-\frac{1}{2})(-\frac{1}{2},\infty)$
No max or min