HW. 5.1 Ans

1. a) $f(x)=x^2+2x-1$ is cont. and differentiable on $[0,1]$. MVT OK.

b) $f'(x)=2x+2$

$$2c+2=\frac{f(1)-f(0)}{1-0}$$
$$2c+2=\frac{2-(-1)}{1}$$
$$2c+2=3$$
$$c=\frac{1}{2}$$

2. a) $f(x)=x^{2/3}$ is continuous and differentiable on $[0,1]$ and $(0,1)$ respectively. MVT ok (we luck out because the differentiable interval does not include $x=0$).

$$f'(x)=\frac{2}{3}x^{-1/3}=\frac{2}{3\sqrt[3]{x}} \qquad \frac{2}{3\sqrt[3]{c}}=\frac{f(1)-f(0)}{1-0}$$

$$\frac{2}{3\sqrt[3]{c}}=\frac{1-0}{1-0}$$
$$\frac{2}{3\sqrt[3]{c}}=1$$
$$2=3\sqrt[3]{c}$$
$$\frac{2}{3}=\sqrt[3]{c}$$
$$\frac{8}{27}=c$$

3. a) $f(x)=x^{1/3}$ is continuous on $[-1,1]$ but not differentiable on $(-1,1)$. MVT not ok. (Note: $f'(x)=\frac{1}{3}x^{-2/3}=\frac{1}{3\sqrt[3]{x^2}}$, vertical tangent when $x=0$).

b) ———

4. a) $f(x)=|x-1|$ is continuous on $[0,4]$ but not differentiable on $(0,4)$. (Note: a corner is at $x=1$). MVT not ok. b) ———

5. a) $f(x)=\sin^{-1}(x)$ is continuous on $[-1,1]$ and differentiable on $(-1,1)$. MVT ok. (Note: $\sin^{-1}(-1)=-\frac{\pi}{2} \longrightarrow \sin^{-1}(1)=\frac{\pi}{2}$)

b) $f'(x)=\frac{1}{\sqrt{1-x^2}}$

$$\frac{1}{\sqrt{1-c^2}}=\frac{f(1)-f(-1)}{1-(-1)}$$
$$\frac{1}{\sqrt{1-c^2}}=\frac{\frac{\pi}{2}-(-\frac{\pi}{2})}{1+1}$$
$$\frac{1}{\sqrt{1-c^2}}=\frac{\pi}{2}$$

$$2=\pi\sqrt{1-c^2}$$
$$\frac{2}{\pi}=\sqrt{1-c^2}$$
$$-\left(\frac{4}{\pi^2}-1\right)=c^2$$
$$c=\sqrt{-\frac{4}{\pi^2}+1}$$

6. a) $f(x) = \ln(x-1)$ is continuous on $[2,4]$ and differentiable on $(2,4)$.

MVT ok.

b) $f'(x) = \frac{1}{x-1}$

$$\frac{1}{c-1} = \frac{f(4)-f(2)}{4-2}$$
$$\frac{1}{c-1} = \frac{\ln(3)-\ln(1)}{2}$$
$$\frac{1}{c-1} = \frac{\ln(3)}{2}$$
$$c-1 = \frac{2}{\ln(3)}$$
$$c = \frac{2}{\ln(3)} + 1$$

7. a) $f(x) = \begin{cases} \cos(x), & 0 \le x < \pi/2 \\ \sin(x), & \pi/2 \le x \le \pi \end{cases}$ is not continuous on $[0,\pi]$ nor differentiable on $(0,\pi)$. (Note: there is a discontinuity at $x = \frac{\pi}{2}$, $\lim_{x\to\frac{\pi}{2}^-} f(x) \ne \lim_{x\to\frac{\pi}{2}^+} f(x)$)

b) —

8. $f(x) = \begin{cases} \sin^{-1}(x) & -1 \le x < 1 \\ x/2 + 1 & 1 \le x \le 3 \end{cases}$

$\sin^{-1}(1) = \pi/2$

$\frac{1}{2} + 1 = 1.5$

$f(x)$ is not continuous on $[-1,3]$ nor differentiable on $(-1,3)$.

(Note: there is a discontinuity at $x = 1$.

9. a) $f(x) = x + \frac{1}{x}$ $\quad \frac{f(2)-f(0.5)}{2-(0.5)} = \frac{2.5-2.5}{1.5} = 0$ ← slope $\quad$ Secant: $y - 2.5 = 0(x-2)$

$f(2) = 2.5$ ← point

b) $f'(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$

$$1 - \frac{1}{x^2} = 0$$
$1 = \frac{1}{x^2} \rightarrow x = \pm 1 \rightarrow x = 1$ on the interval

$f(1) = 2$: T: $y - 2 = 0(x-2)$

10. a) $f(x) = \sqrt{x-1}$

$$\frac{f(3)-f(1)}{3-1} = \frac{\sqrt{2}-0}{2} = \frac{\sqrt{2}}{2}$$

$f(1) = 0$ secant: $y - 0 = \frac{\sqrt{2}}{2}(x-1)$

b) $f'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}}$

$$\frac{1}{2\sqrt{x-1}} = \frac{\sqrt{2}}{2}$$
$$2\sqrt{x-1} = \frac{2}{\sqrt{2}}$$
$$\sqrt{x-1} = \frac{1}{\sqrt{2}}$$
$$x - 1 = \frac{1}{2}$$

$x = \frac{3}{2}$

$f\left(\frac{3}{2}\right) = \frac{\sqrt{2}}{2}$

T: $y - \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\left(x - \frac{3}{2}\right)$