# Homework

- Choose any THREE FRQs to complete.
- Answers will be online with videos.

2012 #6

## Question 6

For $0 \le t \le 12$, a particle moves along the $x$-axis. The velocity of the particle at time $t$ is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time $t$. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \;\Rightarrow\; t = 3,\ 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

2 : { 1 : considers $v(t) = 0$; 1 : interval }

(b) $\int_0^6 |v(t)|\, dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$$

$$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

3 : { 1 : $a(t)$; 2 : conclusion with reason }

(d)
$$\begin{aligned} x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4 \\ &= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right] \\ &= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

3 : { 1 : antiderivative; 1 : uses initial condition; 1 : answer }

# 2005 Form B #3

A particle moves along the $x$-axis so that its velocity $v$ at time $t$, for $0 \le t \le 5$, is given by $v(t) = \ln\left(t^2 - 3t + 3\right)$. The particle is at position $x = 8$ at time $t = 0$.

(a) Find the acceleration of the particle at time $t = 4$.

(b) Find all times $t$ in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \le t \le 5$, does the particle travel to the left?

(c) Find the position of the particle at time $t = 2$.

(d) Find the average speed of the particle over the interval $0 \le t \le 2$.

(a) $a(4) = v'(4) = \frac{5}{7}$

1 : answer

(b) $v(t) = 0$

$t^2 - 3t + 3 = 1$

$t^2 - 3t + 2 = 0$

$(t-2)(t-1) = 0$

$t = 1, 2$

$v(t) > 0$ for $0 < t < 1$

$v(t) < 0$ for $1 < t < 2$

$v(t) > 0$ for $2 < t < 5$

The particle changes direction when $t = 1$ and $t = 2$.
The particle travels to the left when $1 < t < 2$.

3 : { 1 : sets $v(t) = 0$; 1 : direction change at $t = 1, 2$; 1 : interval with reason }

(c) $s(t) = s(0) + \int_0^t \ln\left(u^2 - 3u + 3\right) du$

$s(2) = 8 + \int_0^2 \ln\left(u^2 - 3u + 3\right) du$

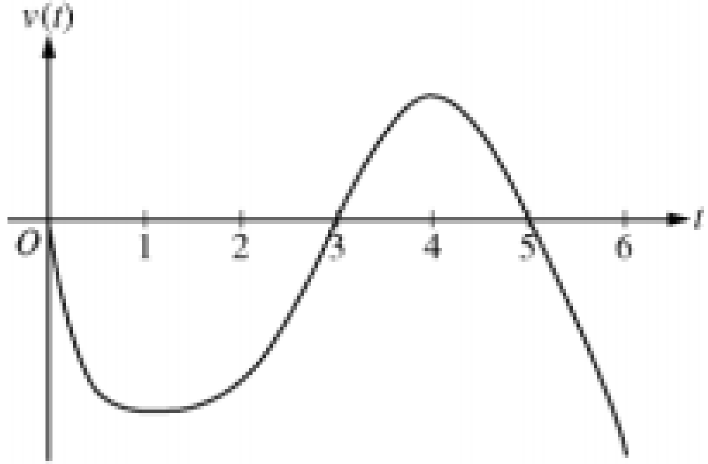
$= 8.368$ or $8.369$

3 : { 1 : $\int_0^2 \ln\left(u^2 - 3u + 3\right) du$; 1 : handles initial condition; 1 : answer }

(d) $\frac{1}{2}\int_0^2 |v(t)|\, dt = 0.370$ or $0.371$

2 : { 1 : integral; 1 : answer }

2008 #4



Graph of $v$

A particle moves along the $x$-axis so that its velocity at time $t$, for $0 \le t \le 6$, is given by a differentiable function $v$ whose graph is shown above. The velocity is 0 at $t = 0$, $t = 3$, and $t = 5$, and the graph has horizontal tangents at $t = 1$ and $t = 4$. The areas of the regions bounded by the $t$-axis and the graph of $v$ on the intervals $[0, 3]$, $[3, 5]$, and $[5, 6]$ are 8, 3, and 2, respectively. At time $t = 0$, the particle is at $x = -2$.

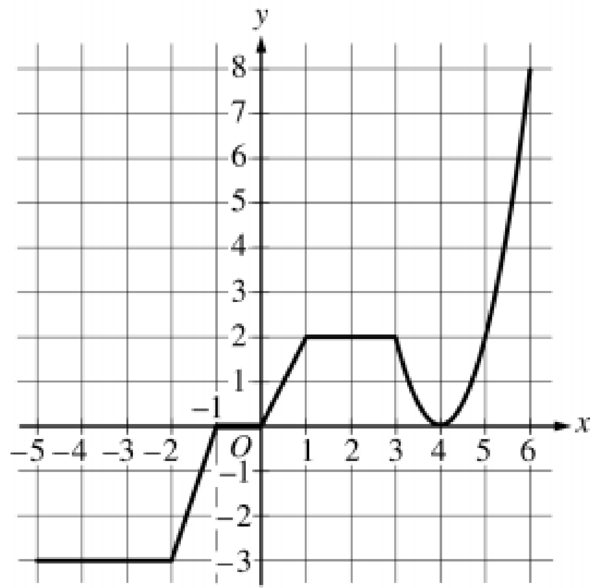
(a) For $0 \le t \le 6$, find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.

(b) For how many values of $t$, where $0 \le t \le 6$, is the particle at $x = -8$ ? Explain your reasoning.

(c) On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

(d) During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

| | |
|---|---|
| (a) Since $v(t) < 0$ for $0 < t < 3$ and $5 < t < 6$, and $v(t) > 0$ for $3 < t < 5$, we consider $t = 3$ and $t = 6$. $$x(3) = -2 + \int_0^3 v(t)\,dt = -2 - 8 = -10$$ $$x(6) = -2 + \int_0^6 v(t)\,dt = -2 - 8 + 3 - 2 = -9$$ Therefore, the particle is farthest left at time $t = 3$ when its position is $x(3) = -10$. | 3 : { 1 : identifies $t = 3$ as a candidate; 1 : considers $\int_0^6 v(t)\,dt$; 1 : conclusion |
| (b) The particle moves continuously and monotonically from $x(0) = -2$ to $x(3) = -10$. Similarly, the particle moves continuously and monotonically from $x(3) = -10$ to $x(5) = -7$ and also from $x(5) = -7$ to $x(6) = -9$. By the Intermediate Value Theorem, there are three values of $t$ for which the particle is at $x(t) = -8$. | 3 : { 1 : positions at $t = 3$, $t = 5$, and $t = 6$; 1 : description of motion; 1 : conclusion |
| (c) The speed is decreasing on the interval $2 < t < 3$ since on this interval $v < 0$ and $v$ is increasing. | 1 : answer with reason |
| (d) The acceleration is negative on the intervals $0 < t < 1$ and $4 < t < 6$ since velocity is decreasing on these intervals. | 2 : { 1 : answer; 1 : justification |




Graph of $g$

3. The graph of the continuous function $g$, the derivative of the function $f$, is shown above. The function $g$ is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$ ?

(b) Evaluate $\int_1^6 g(x)\, dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of $f$ both increasing and concave up? Give a reason for your answer.

(d) Find the $x$-coordinate of each point of inflection of the graph of $f$. Give a reason for your answer.

(a) $f(-5) = f(1) + \int_1^{-5} g(x)\,dx = f(1) - \int_{-5}^{1} g(x)\,dx$

$= 3 - \left(-9 - \frac{3}{2} + 1\right) = 3 - \left(-\frac{19}{2}\right) = \frac{25}{2}$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $\int_1^6 g(x)\,dx = \int_1^3 g(x)\,dx + \int_3^6 g(x)\,dx$

$= \int_1^3 2\,dx + \int_3^6 2(x-4)^2\,dx$

$= 4 + \left[\frac{2}{3}(x-4)^3\right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left(-\frac{2}{3}\right) = 10$

$3 : \begin{cases} 1 : \text{split at } x = 3 \\ 1 : \text{antiderivative of } 2(x-4)^2 \\ 1 : \text{answer} \end{cases}$

(c) The graph of $f$ is increasing and concave up on $0 < x < 1$ and $4 < x < 6$ because $f'(x) = g(x) > 0$ and $f'(x) = g(x)$ is increasing on those intervals.

$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(d) The graph of $f$ has a point of inflection at $x = 4$ because $f'(x) = g(x)$ changes from decreasing to increasing at $x = 4$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$

| $t$ (minutes) | 0 | 2 | 5 | 9 | 10 |
|---|---|---|---|---|---|
| $H(t)$ (degrees Celsius) | 66 | 60 | 52 | 44 | 43 |

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function $H$ for $0 \le t \le 10$, where time $t$ is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time $t$ are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time $t$ is modeled by a differentiable function $B$ for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

| | |
|---|---|
| (a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$<br>$= \dfrac{52 - 60}{3} = -2.666$ or $-2.667$ degrees Celsius per minute | 1 : answer |
| (b) $\dfrac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.<br>$\dfrac{1}{10}\int_0^{10} H(t)\,dt \approx \dfrac{1}{10}\left(2 \cdot \dfrac{66+60}{2} + 3 \cdot \dfrac{60+52}{2} + 4 \cdot \dfrac{52+44}{2} + 1 \cdot \dfrac{44+43}{2}\right)$<br>$= 52.95$ | 3 : { 1 : meaning of expression<br>1 : trapezoidal sum<br>1 : estimate |
| (c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$<br>The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes. | 2 : { 1 : value of integral<br>1 : meaning of expression |
| (d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$<br>The biscuits are 8.817 degrees Celsius cooler than the tea. | 3 : { 1 : integrand<br>1 : uses $B(0) = 100$<br>1 : answer |