Unit 8 Day 3
Accumulation and Definite Integrals in Context

Example: A pipeline company manufactures pipe that sells for $100 per meter. The cost of manufacturing a portion of the pipe varies with its distance from the beginning of the pipe. The company reports that the cost to produce a portion of the pipe that is x meters from the beginning of the pipe is C (x) dollars per meter. The company has already made $1000 from selling this pipe. Write an equation that would model the profit earned by the company after selling x meters of pipe. (Note: Profit is defined as the difference between the amount of money received by the company for selling the pipe and the amount it costs to manufacture the pipe.)

In the scenario below, match the equation with its interpretation. Label the equations with important elements of your interpretation (equivalent expressions, calculus vocab, etc.).

A cup of boiling water is removed from a microwave and set on the counter to cool. The initial temperature of the water is 100 degrees Celsius. The temperature of the water T, measured in degrees Celsius, as a function of time t, measured in minutes, is given by $T(t)$.

_________ 1. $T'(4)$

_________ 2. $100 + \int_0^k T'(t)dt$

_________ 3. $\frac{1}{4}\int_0^4 T(t)dt$

_________ 4. $\frac{1}{4}\int_0^4 T'(t)dt$

_________ 5. $\int_0^4 T'(t)dt$

_________ 6. $T(4)$

_________ 7. $100 + \int_0^k T'(t)dt = 85$

_________ 8. $\frac{1}{k}\int_0^k T(t)dt = 85$

A. An equation that can be solved to find k, the time, in minutes, when the ~~average~~ temperature of the water is 85 degrees Celsius

B. The rate of change in the temperature of the water, in degrees Celsius per minute, at $t = 4$ minutes

C. The average temperature of the water, in degrees Celsius, between $t = 0$ and $t = 4$ minutes

D. The difference in the temperature of the water, in degrees Celsius, between $t = 0$ and $t = 4$ minutes

E. The temperature of the water, in degrees Celsius, at $t = 4$ minutes

F. The average rate of change in the temperature of the water, in degrees Celsius

G. The temperature of the water, in degrees Celsius, at $t = k$ minutes

H. An equation that can be solved to determine the time k, in minutes, when the average temperature of the water, in degrees Celsius, was 85, over the time interval from $t = 0$ to $t = k$ minutes

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

2 : { 1 : $G'(5)$
1 : interpretation with units

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

2 : { 1 : integral
1 : answer

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

2 : { 1 : compares $G(5)$ to 100
1 : conclusion

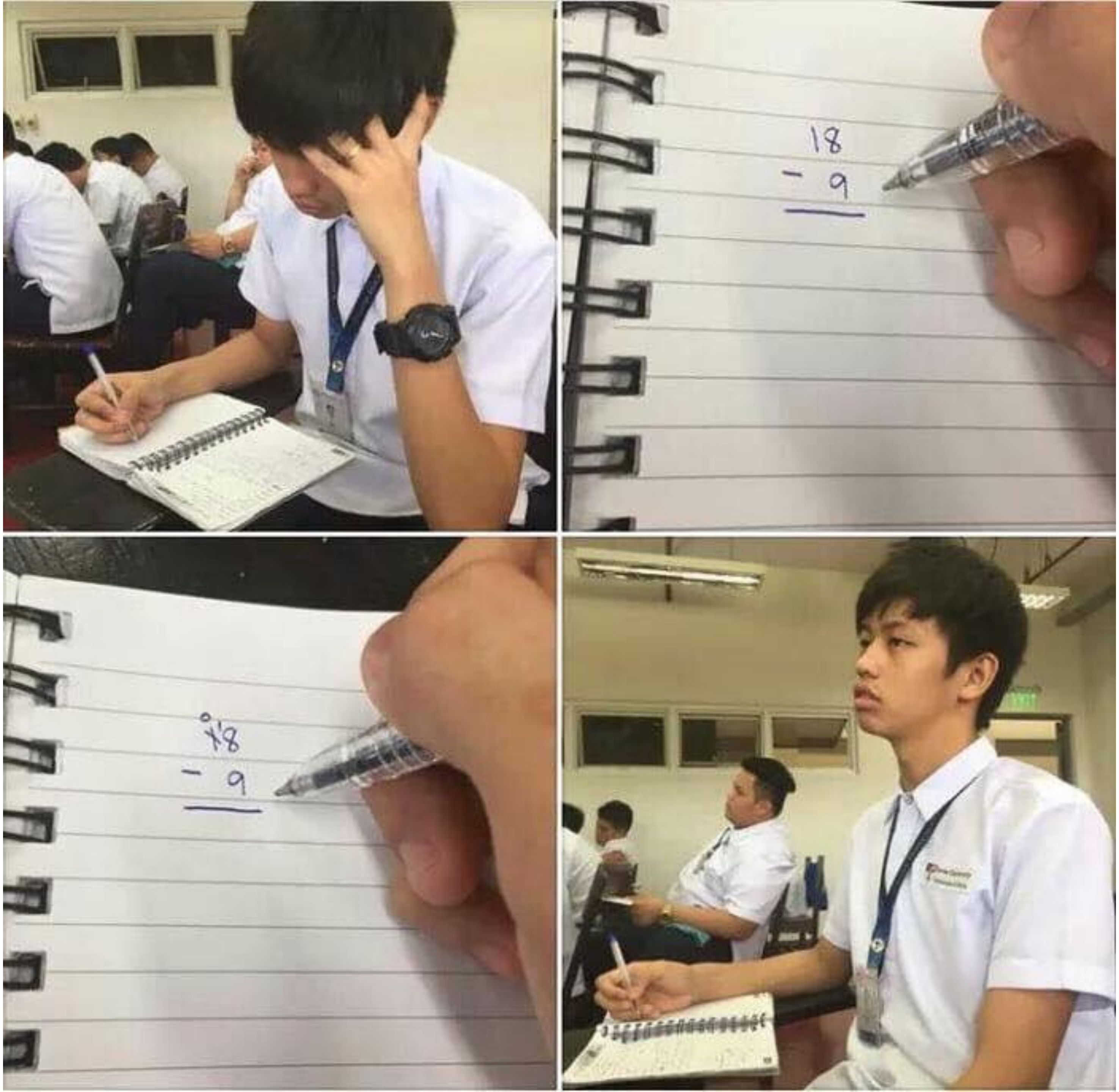
(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds.$

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

3 : { 1 : considers $A'(t) = 0$
1 : answer
1 : justification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1. Fish enter a lake at a rate modeled by the function E given by $E(t) = 20 + 15\sin\left(\frac{\pi t}{6}\right)$. Fish leave the lake at a rate modeled by the function L given by $L(t) = 4 + 2^{0.1t^2}$. Both $E(t)$ and $L(t)$ are measured in fish per hour, and t is measured in hours since midnight ($t = 0$).

(a) How many fish enter the lake over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)? Give your answer to the nearest whole number.

(b) What is the average number of fish that leave the lake per hour over the 5-hour period from midnight ($t = 0$) to 5 A.M. ($t = 5$)?

(c) At what time t, for $0 \le t \le 8$, is the greatest number of fish in the lake? Justify your answer.

(d) Is the rate of change in the number of fish in the lake increasing or decreasing at 5 A.M. ($t = 5$)? Explain your reasoning.

(a) $\int_0^5 E(t)\,dt = 153.457690$

To the nearest whole number, 153 fish enter the lake from midnight to 5 A.M.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(b) $\frac{1}{5-0}\int_0^5 L(t)\,dt = 6.059038$

The average number of fish that leave the lake per hour from midnight to 5 A.M. is 6.059 fish per hour.

$2: \begin{cases} 1: \text{integral} \\ 1: \text{answer} \end{cases}$

(c) The rate of change in the number of fish in the lake at time t is given by $E(t) - L(t)$.

$E(t) - L(t) = 0 \Rightarrow t = 6.20356$

$E(t) - L(t) > 0$ for $0 \le t < 6.20356$, and $E(t) - L(t) < 0$ for $6.20356 < t \le 8$. Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

— OR —

Let $A(t)$ be the change in the number of fish in the lake from midnight to t hours after midnight.

$A(t) = \int_0^t (E(s) - L(s))\,ds$

$A'(t) = E(t) - L(t) = 0 \Rightarrow t = C = 6.20356$

t	$A(t)$
0	0
C	135.01492
8	80.91998

Therefore the greatest number of fish in the lake is at time $t = 6.204$ (or 6.203).

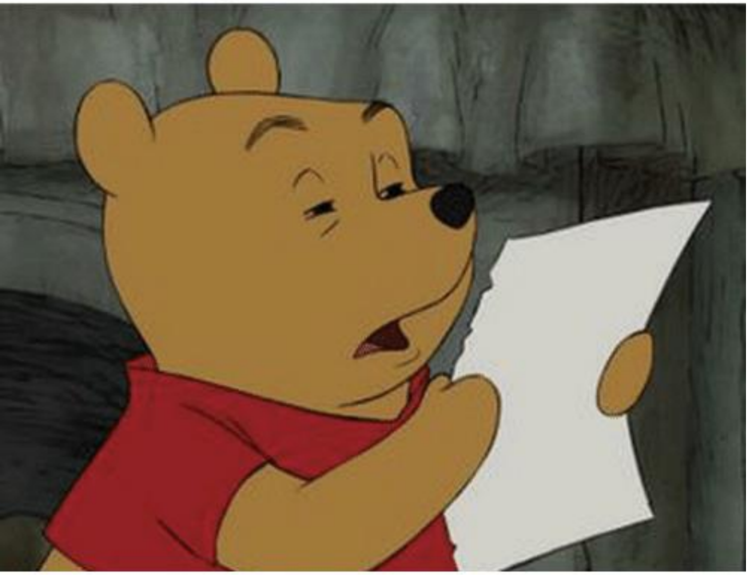
$3: \begin{cases} 1: \text{sets } E(t) - L(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(d) $E'(5) - L'(5) = -10.7228 < 0$

Because $E'(5) - L'(5) < 0$, the rate of change in the number of fish is decreasing at time $t = 5$.

$2: \begin{cases} 1: \text{considers } E'(5) \text{ and } L'(5) \\ 1: \text{answer with explanation} \end{cases}$

Me staring at the Calc AB FRQ like the fish are going to magically come out of the lake and give me the answers



Form O really L(t)ing us down

Fish: actually I want to stay in the lake right n-
L(t):

rip calc fish :(

Fish when it's time to leave the lake

2. When a certain grocery store opens, it has 50 pounds of bananas on a display table. Customers remove bananas from the display table at a rate modeled by

$$f(t) = 10 + (0.8t)\sin\left(\frac{t^3}{100}\right) \text{ for } 0 < t \le 12,$$

where $f(t)$ is measured in pounds per hour and t is the number of hours after the store opened. After the store has been open for three hours, store employees add bananas to the display table at a rate modeled by

$$g(t) = 3 + 2.4\ln\left(t^2 + 2t\right) \text{ for } 3 < t \le 12,$$

where $g(t)$ is measured in pounds per hour and t is the number of hours after the store opened.

(a) How many pounds of bananas are removed from the display table during the first 2 hours the store is open?

(b) Find $f'(7)$. Using correct units, explain the meaning of $f'(7)$ in the context of the problem.

(c) Is the number of pounds of bananas on the display table increasing or decreasing at time $t = 5$? Give a reason for your answer.

(d) How many pounds of bananas are on the display table at time $t = 8$?

(a) $\int_0^2 f(t)\,dt = 20.051175$

20.051 pounds of bananas are removed from the display table during the first 2 hours the store is open.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $f'(7) = -8.120$ (or -8.119)

After the store has been open 7 hours, the rate at which bananas are being removed from the display table is decreasing by 8.120 (or 8.119) pounds per hour per hour.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{meaning} \end{cases}$

(c) $g(5) - f(5) = -2.263103 < 0$

Because $g(5) - f(5) < 0$, the number of pounds of bananas on the display table is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{considers } f(5) \text{ and } g(5) \\ 1 : \text{answer with reason} \end{cases}$

(d) $50 + \int_3^8 g(t)\,dt - \int_0^8 f(t)\,dt = 23.347396$

23.347 pounds of bananas are on the display table at time $t = 8$.

$3 : \begin{cases} 2 : \text{integrals} \\ 1 : \text{answer} \end{cases}$

When the customers start taking bananas at a faster rate than you can stock them
#apcalc

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2 : \begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is

$30 + \int_0^t [R(x) - D(x)]\,dx.$

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3 : \begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

   (a) Estimate $R'(2)$. Show the work that leads to your answer. Indicate units of measure.

   (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.

   (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.

   (d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

(a) $R'(2) \approx \dfrac{R(3)-R(1)}{3-1} = \dfrac{950-1190}{3-1} = -120 \text{ liters/hr}^2$

$2: \begin{cases} 1: \text{estimate} \\ 1: \text{units} \end{cases}$

(b) The total amount of water removed is given by $\int_0^8 R(t)\,dt$.

$$\begin{aligned}\int_0^8 R(t)\,dt &\approx 1\cdot R(0) + 2\cdot R(1) + 3\cdot R(3) + 2\cdot R(6) \\ &= 1(1340) + 2(1190) + 3(950) + 2(740) \\ &= 8050 \text{ liters}\end{aligned}$$

This is an overestimate since R is a decreasing function.

$3: \begin{cases} 1: \text{left Riemann sum} \\ 1: \text{estimate} \\ 1: \text{overestimate with reason} \end{cases}$

(c) $$\begin{aligned}\text{Total} &\approx 50000 + \int_0^8 W(t)\,dt - 8050 \\ &= 50000 + 7836.195325 - 8050 \approx 49786 \text{ liters}\end{aligned}$$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{estimate} \end{cases}$

(d) $W(0) - R(0) > 0$, $W(8) - R(8) < 0$, and $W(t) - R(t)$ is continuous.

Therefore, the Intermediate Value Theorem guarantees at least one time t, $0 < t < 8$, for which $W(t) - R(t) = 0$, or $W(t) = R(t)$.

For this value of t, the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank.

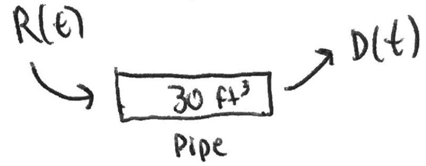
$2: \begin{cases} 1: \text{considers } W(t) - R(t) \\ 1: \text{answer with explanation} \end{cases}$

Homework

- Prepare the FRQs from today.
- For THURSDAY, correct the FRQs from the semester 1 exam. Please redo each FRQ on a separate piece of paper if you missed ANY points on that FRQ.
  - Redo the WHOLE FRQ for practice
  - You can ask me questions and get help from others.
  - You will turn in your test AND corrections
  - You do not need to rewrite a question, etc.
- If you have not take the Unit 7 test, let’s talk.

2015 #1

a)



Into pipe: R(t)

$\int_0^8 R(t)dt = 76.570 \text{ ft}^3$

b) $W(t) = 30 + \int_0^t R(x)dx - \int_0^t D(x)\,dx$

$W'(t) = R(t) - D(t)$

$R(3) - D(3) = -0.313632$ this is < 0. That means the amount of water in the pipe is decreasing because $D(3) > R(3)$ at t=3 hrs

c) $W(t) = 30 + \int_0^t R(x)dx - \int_0^t D(x)dx$

Absolute min may occur at EPs or CPs.

	t	W(t)
EP	0	30
CP	3.271658	27.964561
EP	8	48.543686

Amount of water is at a minimum when t = 3.271 hrs or 3.272 hrs

CP when $W'(t) = 0$

$W'(t) = R(t) - D(t)$

$R(t) - D(t) = 0$

$R(t) = D(t)$

Graph to find intersection

$t = 0,\ 3.271658$

d) $30 + \int_0^w R(t)dt - \int_0^w D(t)dt = 50$

2016 #1

a) $R'(2)$ estimate

R(t) in the table (1, 1190) (3, 950)

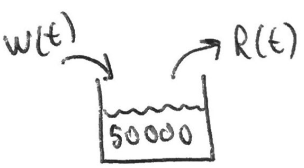
$$\frac{950-1190 \text{ liters/hr}}{3-1 \text{ hr}} = -120 \text{ liters/hr}^2$$

b) <u>Not</u> even intervals

$1(1340) + 2(1190) + 3(950) + 2(740) \approx 8050$ liters

hr(liter/hr) ⇒ liters

This amount is an overestimate b/c R(t) is decreasing and Left Riemann sum is used.

c) 

$$A(8) = 50000 + \int_0^8 w(t)\,dt - \int_0^8 R(t)\,dt$$
$$= 50\,000 + 7836.195 - 8050 \approx 49786 \text{ liters}$$

d) $w(0) = 2000e^0 = 2000$ liters/hr $\quad w(8) = 2000e^{-8^2/20} = 81.524$ liters/hr

$R(0) = 1340$ liters/hr $\quad R(8) = 700$ liters/hr

At the start, $w(0) > R(0)$; then, by $t=8$, $w(8) < R(8)$; Since both both $w(t)$ and $R(t)$ are differentiable and continuous, according to the IVT (Intermediate Value Theorem) $w(t) = R(t)$ somewhere $0 < t < 8$. This means that the rate water is being pumped in will equal the rate it is being removed at that t.