HW 7.3

Pg. 364 #1-10

1. $\frac{dy}{dx} = \frac{x}{y}$ $(1,2)$

$\int y\,dy = \int x\,dx$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + C$ at $(1,2)$

$2 = \frac{1}{2} + C$

$\frac{3}{2} = C$

$\frac{1}{2}y^2 = \frac{1}{2}x^2 + \frac{3}{2}$

$y^2 = x^2 + 3$

$y = \pm\sqrt{x^2+3}$

$\boxed{y = \sqrt{x^2+3}}$ on all real #s

2. $\frac{dy}{dx} = -\frac{x}{y}$ $(4,3)$

$\int y\,dy = \int -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$ at $(4,3)$

$\frac{9}{2} = -8 + C$

$\frac{25}{2} = C$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + \frac{25}{2}$

$y^2 = -x^2 + 25$

$y = \pm\sqrt{-x^2+25}$

$\boxed{y = \sqrt{-x^2+25}}$ on $[-5,5]$

3. $\frac{dy}{dx} = \frac{y}{x}$ $(2,2)$

$\int \frac{1}{y}\,dy = \int \frac{1}{x}\,dx$

$e^{\ln|y|} = e^{\ln|x| + C}$

$|y| = e^{\ln|x|+C}$

$|y| = e^{\ln|x|} \cdot e^{C}$

$y = Ce^{\ln|x|}$

$y = C|x|$ at $(2,2)$

$2 = 2C$

$C = 1$

$\boxed{y = |x| \text{ or } y = x \text{ on } (0,\infty)}$

4. $\frac{dy}{dx} = 2xy$ $(0,3)$

$\int \frac{1}{y}\,dy = \int 2x\,dx$

$e^{\ln|y|} = e^{x^2 + C}$

$|y| = e^{x^2+C}$

$|y| = e^{x^2} \cdot e^{C}$

$y = Ce^{x^2}$ at $(0,3)$

$3 = C$

$\boxed{y = 3e^{x^2}}$ on all real #s

5. $\frac{dy}{dx} = (y+5)(x+2)$ $(0,1)$

$\int \frac{1}{y+5}\,dy = \int (x+2)\,dx$

$e^{\ln|y+5|} = e^{\frac{1}{2}x^2+2x+C}$

$y+5 = Ce^{\frac{1}{2}x^2+2x}$ at $(0,1)$

$6 = C$

$\rightarrow y+5 = 6e^{\frac{1}{2}x^2+2x}$

$\boxed{y = 6e^{\frac{1}{2}x^2+2x} - 5}$ on all real #s

6. $\frac{dy}{dx} = \cos^2(y)$ $(0,0)$

$\frac{1}{\cos^2(y)}\,dy = 1\,dx$

$\int \sec^2(y)\,dy = \int 1\,dx$

$\tan(y) = x + C$ at $(0,0)$

$0 = C$

$\rightarrow \tan(y) = x$

$\boxed{y = \tan^{-1}(x)}$ except $\frac{\pi}{2} + \pi n = x,\ n \in \mathbb{Z}$

7. $\frac{dy}{dx} = \cos(x)e^{y+\sin(x)}$ $(0,0)$

$\frac{dy}{dx} = \cos(x)e^{y} \cdot e^{\sin(x)}$

$\frac{1}{e^y}\,dy = \cos(x)e^{\sin(x)}\,dx$

$\int e^{-y} = \int \cos(x)e^{\sin(x)}\,dx$

u-sub both sides and $u = -y$ and $u = \sin(x)$

$-e^{-y} = e^{\sin(x)} + C$ $(0,0)$

$-1 = 1 + C$

$-2 = C$

$\rightarrow -e^{-y} = e^{\sin(x)} - 2$

$e^{-y} = -e^{\sin(x)} + 2$

$-y = \ln(-e^{\sin(x)} + 2)$

$\boxed{y = -\ln(-e^{\sin(x)} + 2)}$ on all real #s

8. $\frac{dy}{dx} = e^{x-y}$ $(0,2)$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$\int e^y\,dy = \int e^x\,dx$

$e^y = e^x + C$ at $(0,2)$

$e^2 = 1 + C$

$e^2 - 1 = C$

$\rightarrow e^y = e^x + e^2 - 1$

$\boxed{y = \ln(e^x + e^2 - 1)}$ on all real #s

9. $\frac{dy}{dx} = -2xy^2 \quad (1, \frac{1}{4})$

$\int \frac{1}{y^2} dy = \int -2x\,dx$

$\int y^{-2} dy = \int -2x\,dx$

$-y^{-1} = -x^2 + C \quad (1, \frac{1}{4})$

$-\left(\frac{1}{4}\right)^{-1} = -1 + C$

$-4 = -1 + C$

$-3 = C$

$\rightarrow -y^{-1} = -x^2 - 3$

$y^{-1} = x^2 + 3$

$\boxed{y = \frac{1}{x^2+3}}$ on all real #s

10. $\frac{dy}{dx} = \frac{4\sqrt{y}\ln(x)}{x} \quad (e, 1)$

$\int \frac{1}{\sqrt{y}} dy = \int 4\frac{\ln(x)}{x} dx$

$\int y^{-1/2} dy = 4\int \frac{\ln(x)}{x} dx$ ⇢ $u = \ln(x)$, $\frac{du}{dx} = \frac{1}{x}$, $x\,du = dx$

$\int y^{-1/2} dy = 4\int u\,du$

$2y^{1/2} = 4 \cdot \frac{1}{2}u^2 + C$

$2y^{1/2} = 2(\ln(x))^2 + C \quad (e, 1)$

$2 = 2 + C$

$C = 0$

$\rightarrow 2y^{1/2} = 2(\ln(x))^2$

$y^{1/2} = (\ln(x))^2$

$\boxed{y = (\ln(x))^4}$ on $(0, \infty)$