Selected Answers

Homework

For each function, find the intervals of increasing and decreasing. Then categorize the local extrema as maxima, minima, or neither.

1. $f(x) = x^3 - 3x^2 - 24x + 2$
2. $f(x) = 2x^3 + 3x^2 - 36x + 5$
3. $f(x) = x^3 - 12x + 5$
4. $f(x) = 3x^4 - 4x^3 - 36x^2 - 5$

14. $f(x) = \dfrac{5x}{(x+3)^3}$
15. $f(x) = \dfrac{x^2}{(x-4)^3}$
16. $f(x) = \dfrac{x^2}{x^2+1}$

(Note: Domain $= \mathbb{R}$.)

1. $f'(x) = 3x^2 - 6x - 24 = 3(x^2 - 2x - 8) = 3(x-4)(x+2)$ CP: $x = 4, -2$

$f'(x)$	$\nearrow$ ⊕	-2 $\searrow$ ⊖	4 $\nearrow$ ⊕
3	+	+	+
$(x-4)$	−	−	+
$(x+2)$	−	+	+

f(x) is increasing on $(-\infty, -2)$ and $(4, \infty)$ b/c $f'(x)$ is positive.
f(x) is decreasing on $(-2, 4)$ b/c $f'(x)$ is negative
$f'(x)$ has a local min at $x = 4$ b/c $f'(x)$ changes − to +.
$f'(x)$ has a local max at $x = -2$ b/c $f'(x)$ changes + to −.

14. $f'(x) = \dfrac{(x+3)^3(5) - 5x \cdot 3(x+3)^2}{(x+3)^6} = \dfrac{5(x+3)^3 - 15x(x+3)^2}{(x+3)^6} = \dfrac{5(x+3) - 15x}{(x+3)^4} = \dfrac{-10x + 15}{(x+3)^4}$

CP: $x = -3$ and $x = \frac{3}{2}$

$f'(x)$	$\nearrow$ ⊕	-3 $\nearrow$ ⊕	$3/2$ $\searrow$ ⊖
$-10x + 15$	+	+	−
$(x+3)^4$	+	+	+

f(x) is increasing on $(-\infty, -3)$ and $(-3, 3/2)$ b/c $f'(x)$ is positive.
f(x) is decreasing on $(\frac{3}{2}, \infty)$ b/c $f'(x)$ is negative
f(x) has a local max at $x = 3/2$ b/c $f'(x)$ changes + to −
At $x = -3$, there is neither a local min nor local max b/c $f'(x)$ does not change sign.

HW: Pg. 220 #1-6 (do not need to support graphically), #23-24

5.3 Selected Answers

Pg. 220

#4 $y = x e^{1/x}$

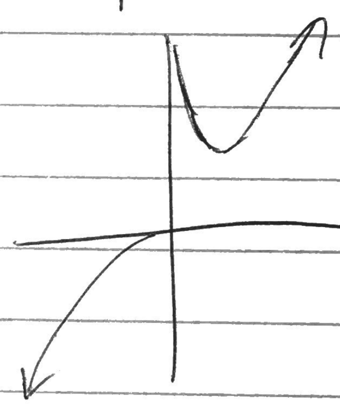
$$\frac{dy}{dx} = x e^{1/x}\left(\frac{-1}{x^2}\right) + e^{1/x} = -\frac{1}{x} e^{1/x} + e^{1/x} = e^{1/x}\left(-\frac{1}{x} + 1\right)$$

CP: $x = 0$ and $x = 1$

	↗ +	0 ↘ −	1 ↗ +
$e^{1/x}$	+	+	+
$(-\frac{1}{x}+1)$	+	−	+

Local max at $x = 0$, local min at $x = 1$

No abs extrema: y keeps inc and dec



#24 a) f is increasing $(-2,0)$ and $(0,2)$ b/c $f'(x)$ is positive

b) f is decreasing $(-\infty,-2)$ and $(2,\infty)$ b/c $f'(x)$ is negative

c) f has a local min at $x=-2$ b/c $f'(x)$ changes − to +

f has a local max at $x=2$ b/c $f'(x)$ changes + to −

at $x=0$, it is neither a max nor min on f b/c $f'(x)$ does not change sign.