HWK Assigned 10/25 du 10/28

1. Critical #'s : $f'(x)=0$ or $f'(x)$ UD. $f'(x)=3x^2+6x-24$

$f'(x)=3x^2+6x-24=0$

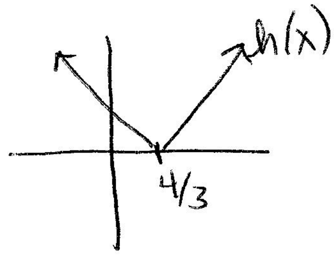
$x^2+2x-8=0$

$(x+4)(x-2)=0$

$\boxed{x=2,\ x=4}$

$f'(x)$ UD nowhere!

3. $h(x)=|3x-4|$



$h'(x)=0$ nowhere!

$h'(x)$ UD at $\boxed{x=4/3}$

2. $g(t)=3t^4+4t^3-6t^2$

$g'(t)=12t^3+12t^2-12t$

$g'(t)=12t(t^2+t-1)=0$

when $\boxed{t=0}$ $t^2+t-1=0$

$t=\dfrac{-1\pm\sqrt{1^2-4(1)(-1)}}{2(1)}$

$t=\dfrac{-1\pm\sqrt{1+4}}{2}$

$\boxed{t=\dfrac{-1\pm\sqrt{5}}{2}}$

$g'(t)$ UD nowhere!

5. $g(t)=2t^3+2t^2-12t+4$ on $[0,2]$

same critical pts as #4

t	$g(t)$
0	$g(0)=4$
1	$g(1)=-3$
2	$g(2)=8$

absolute max: $y=8$ at $x=2$

absolute min: $y=-3$ at $x=1$

4. $g(t)=2t^3+3t^2-12t+4$ on $[-4,2]$

$g'(t)=6t^2+6t-12=6(t^2+t-2)$

$g'(t)=6(t+2)(t-1)=0$ at $t=-2, t=1$

$g'(t)$ U/D nowhere

t	$g(t)$
-4	$g(-4)=2(-4)^3+3(4)^2-12(-4)+4=-28$
-2	$g(-2)=2(-2)^3+3(-2)^2-12(-2)+4=24$
1	$g(1)=2(1)^3+3(1)^2-12(1)+4=-3$
2	$g(2)=2(2^3)+3(2^2)-12(2)+4=8$

absolute max: $y=24$ at $x=-2$

" min: $y=-28$ at $x=-4$

6. $f(x)=x^4-2x^2$ on $[-2,3]$

$f'(x)=4x^3-4x=4x[x^2-1]=4x(x-1)(x+1)$

$f'(x)=0$ at $x=0, x=1, x=-1$

$f'(x)$ UD nowhere

x	$f(x)$
-2	$f(-2)=(-2)^4-2(-2)^2=8$
-1	$f(-1)=(-1)^4-2(-1)^2=-1$
0	$f(0)=0$
1	$f(1)=1-2=-1$
3	$f(3)=3^4-2(3^2)=63$

abso. max: $y=63$ at $x=3$

" min: $y=-1$ at $x=-1, x=1$

7. $f(x) = (x^2-1)^3$ on $[-1, 2]$

$f'(x) = 3(x^2-1)^2(2x) = 6x(x^2-1)^2$

$f'(x) = 6x(x^2-1)^2 = 0$ when

$x=0,\ x=1,\ x=-1$

$f'(x)$ U/D nowhere.

x	f(x)
-1	$f(-1) = ((-1)^2-1)^3 = 0$
0	$f(0) = -1$
1	$f(1) = 0$
2	$f(2) = (2^2-1)^3 = 27$

abs max: $y = 27$ at $x=2$
" min: $y = -1$ at $x=0$

8. $y = \dfrac{x}{(x^2+1)}$ on $[-1, 2]$

$y' = \dfrac{(x^2+1)(1) - x(2x)}{(x^2+1)^2}$

$y' = \dfrac{x^2+1-2x^2}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$

$y' = 0$ when $\dfrac{1-x^2}{(x^2+1)^2} = 0$

so $1-x^2 = 0$ at

$\underline{x=1},\ \underline{x=-1}$

y' U/D nowhere

x	y
-1	$y(-1) = \frac{-1}{2}$
1	$y(1) = \frac{1}{2}$
2	$y(2) = \frac{2}{5}$

abs max: $\frac{1}{2}$ at $x=1$
" min: $-\frac{1}{2}$ at $x=-1$

9. $g(x) = \sqrt{1-x^2} = (1-x^2)^{1/2}$

$g'(x) = \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{1-x^2}}$

$g'(x) = 0$ at $\underline{x=0}$ ← makes top $=0$

$g'(x)$ U/D at $\underline{x = 1, -1}$ ← makes bottom $=0$.

x	g(x)
-1	$g(-1) = 0$
0	$g(0) = 1$
1	$g(1) = 0$

abs. max: $y = 1$ at $x=0$
" min: $y = 0$ at $x = 1, -1$

10. $A(t) = 2000 - 10te^{5-t^2/8}$ $[0, 10]$

$A'(t) = -10t\left[e^{5-t^2/8}\right]\left(\frac{-2t}{8}\right) + \left(e^{5-t^2/8}\right)(-10)$

$A'(t) = -10e^{5-t^2/8}\left[\frac{-1t^2}{4} + 1\right] = 0$

when $\frac{-1t^2}{4} + 1 = 0$ $\quad \frac{t^2}{4} = 1$ $\quad t^2 = 4$

$\underline{t = 2, \cancel{-2}}$

not in domain

$-10e^{5-t^2/8} \neq 0$

← always +

$\underline{A'(t)\ \text{U/D never!}}$

t	A(t)
0	$A(0) = 2000$
2	$A(2) = 199.66$
10	$A(10) = 1999.94$

abs. max: $A(0) = 2000$
at $t=0$

abs min: $A(2) = 199.66$
at $t=2$

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10. $f(x)=2x-3x^{2/3}$

$f'(x)=2-2x^{-1/3}=2-\frac{2}{\sqrt[3]{x}}$ $\rightarrow x\neq 0$

$2-\frac{2}{\sqrt[3]{x}}=0$

$2=\frac{2}{\sqrt[3]{x}} \rightarrow \sqrt[3]{x}=1$

$x=1$

CP: $x=0,1$ $\quad f(0)=0 \quad f(1)=-1$

End points: $x=-1,3$ $\quad f(-1)=-5 \quad f(3)=-0.2403$

Abs Max is 0 at $x=0$

Abs Min is -5 at $x=-1$

11. $f(x)=(x+2)^3(x-3)^5$

$f'(x)=(x+2)^3\cdot 5(x-3)^4+3(x+2)^2(x-3)^5$

$f'(x)=(x+2)^2(x-3)^4[5(x+2)+3(x-3)]$

$f'(x)=(x+2)^2(x-3)^4(5x+10+3x-9)$

$f'(x)=(x+2)^2(x-3)^4(8x+1)$

Absolute extremes could be at $x=-2,3,-\frac{1}{8},-5,$ or 5

		-2		$-1/8$		3	
$f'(x)$	$-$		$-$		$+$		$+$
$(x+2)^2$	$+$		$+$		$+$		$+$
$(x-3)^4$	$+$		$+$		$+$		$+$
$(8x+1)$	$-$		$-$		$+$		$+$

$f(x)$ increases $(-1/8,3)$ and $(3,\infty)$

$f(x)$ decreases $(-\infty,-2)$ and $(-2,-1/8)$

5.2 HW

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15. $f(x) = 5x - x^2$

$f'(x) = 5 - 2x$

CP: $5/2$

$f'(x)$	$+$	$5/2$	$-$
$5-x$	$+$		$-$

Local max at $x = 5/2$, $f(5/2) = \frac{25}{4}$

$f(x)$ is increasing $(-\infty, 5/2)$; $f(x)$ is decreasing $(5/2, \infty)$

16. $g(x) = x^2 - x - 12$

$g'(x) = 2x - 1$

CP: $x = \frac{1}{2}$

$f'(x)$	$-$	$1/2$	$+$
$2x-1$	$-$		$+$

Local min at $x = 1/2$ $g(\frac{1}{2}) = \frac{1}{4} - \frac{1}{2} - 12$

$g(x)$ is increasing $(1/2, \infty)$; $g(x)$ is decreasing $(-\infty, 1/2)$

17. $h(x) = \frac{2}{x} = 2x^{-1}$

$h'(x) = -2x^{-2} = \frac{-2}{x^2}$

CP: $x = 0$

$h'(x)$	$-$	0	$-$
-2	$-$		$-$
always $+$ → x^2	$+$		$+$

No local extrema

$h(x)$ is never increasing; $h(x)$ is decreasing $(-\infty, 0)$ and $(0, \infty)$

18. $K(x) = \frac{1}{x^2} = x^{-2}$

$K'(x) = -2x^{-3} = \frac{-2}{x^3}$

CP: $x = 0$

$K'(x)$	$+$	0	$-$
-2	$-$		$-$
x^3	$-$		$+$

No local extrema (would have a max at $x = 0$, but $K(0)$ DNE)

$K(x)$ is increasing $(-\infty, 0)$ and $k(x)$ is decreasing $(0, \infty)$

19. $f(x) = e^{2x}$

$f'(x) = e^{2x} \cdot 2$

CP: none! e^{2x} is always $+$

$f(x)$	$+$
e^{2x}	$+$
2	$+$

$f(x)$ has no local extrema ($f'(x)$ never changes sign)

$f(x)$ is increasing $(-\infty, \infty)$; $f(x)$ never decreases

20. $f(x)=e^{-0.5x}$

$f'(x)=e^{-0.5x}(-0.5)$

CP: None! $e^{-0.5x}$ is always +

$f'(x)$	⊖
-0.5	−
$e^{-0.5x}$	+

No local extrema ($f'(x)$ never changes sign).

$f(x)$ never increases; $f(x)$ decreases $(-\infty,\infty)$

21. $y=4-\sqrt{x+2}=4-(x+2)^{1/2}$

$\frac{dy}{dx}=-\frac{1}{2}(x+2)^{-1/2}=\frac{-1}{2\sqrt{x+2}}$

CP: $x=-2$

dy/dx	DNE	-2 ⊖
-1	−	−
$2\sqrt{x+2}$	DNE	+

Local max at $x=-2$, $y=4$

y never increases; y decreases $(-2,\infty)$

22. $y=x^4-10x^2+9$

$\frac{dy}{dx}=4x^3-20x=4x(x^2-5)=4x(x-\sqrt{5})(x+\sqrt{5})$

CP: $x=0,\sqrt{5},-\sqrt{5}$ $\rightarrow y=-16$

Local min at $x=-\sqrt{5},\sqrt{5}$ $\rightarrow y=-16$

Local max at $x=0$, $y=9$

dy/dx	⊖	$-\sqrt{5}$ ⊕	0 ⊖	$\sqrt{5}$ ⊕
$4x$	−	−	+	+
$x-\sqrt{5}$	−	−	−	+
$x+\sqrt{5}$	−	+	+	+

y is increasing $(-\sqrt{5},0)$ and $(\sqrt{5},\infty)$; y is decreasing $(-\infty,-\sqrt{5})$ and $(0,\sqrt{5})$

23. $f(x)=x\sqrt{4-x}=x(4-x)^{1/2}$

$f'(x)=x\frac{1}{2}(4-x)^{-1/2}(-1)+(4-x)^{1/2}$

$f'(x)=\frac{-x}{2\sqrt{4-x}}+\sqrt{4-x}\cdot\frac{2\sqrt{4-x}}{2\sqrt{4-x}}=\frac{-x}{2\sqrt{4-x}}+\frac{8-2x}{2\sqrt{4-x}}=\frac{8-3x}{2\sqrt{4-x}}=\frac{8-3x}{2\sqrt{4-x}}$

CP: $x=8/3,\ 4$

$f'(x)$	⊕	$8/3$ ⊖	4 DNE
$2-x$	+	−	−
$\sqrt{4-x}$	+	+	DNE

Local max at $x=8/3$, $f(8/3)=\frac{16\sqrt{3}}{9}$ and local min $x=4$, $f(4)=0$

$f(x)$ is increasing $(-\infty,8/3)$; $f(x)$ is decreasing $(8/3,4)$

24. $g(x) = x^{1/3}(x+8) = x^{4/3} + 8x^{1/3}$

$g'(x) = \frac{4}{3}x^{1/3} + \frac{8}{3}x^{-2/3} = \frac{4}{3}x^{-2/3}(x+2) = \frac{4(x+2)}{3\sqrt[3]{x^2}}$

CP: $x = -2, 0$

$g'(x)$	$-$ 0	$+$ 2	$+$
$4(x+2)$	$-$	$+$	$+$
$3\sqrt[3]{x^2}$	$+$	$+$	$+$

Local min at $x=0$, $g(0)=0$

$g(x)$ is increasing $(0,2)$ and $(2,\infty)$

$g(x)$ is decreasing $(-\infty,0)$

25. $h(x) = \frac{-x}{x^2+4}$

$h'(x) = \frac{(x^2+4)(-1) - (-x)(2x)}{(x^2+4)^2} = \frac{-x^2-4+2x^2}{(x^2+4)^2} = \frac{x^2-4}{(x^2+4)^2}$

$h'(x) = \frac{(x-2)(x+2)}{\underbrace{(x^2+4)^2}_{\text{always }+}}$

$h'(x)$	$+$ -2	$-$ 2	$+$
$(x-2)$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$
$(x^2+4)^2$	$+$	$+$	$+$

CP: $x = 2, -2$

$h(-2) = 1/4$

Local max at $x=-2$, local min at $x=2$, $h(2) = -1/4$

$h(x)$ is increasing $(-\infty,-2)$ and $(2,\infty)$; $h(x)$ is decreasing $(-2, 2)$

26. $k(x) = \frac{x}{x^2-4}$

$k'(x) = \frac{(x^2-4)(1) - x(2x)}{(x^2-4)^2} = \frac{x^2-4-2x^2}{[(x-2)(x+2)]^2} = \frac{-x^2-4}{[(x-2)(x+2)]^2} = \frac{-\overbrace{(x^2+4)}^{\text{no zeros}}}{\underbrace{[(x-2)(x+2)]^2}_{\text{always} \geq 0 \text{ b/c } [\ \ ]^2}}$

CP: $x = 2, -2$

$k'(x)$	$-$ -2	$-$ 2	$-$
$-(x^2+4)$	$-$	$-$	$-$
$(x-2)^2$	$+$	$+$	$+$
$(x+2)^2$	$+$	$+$	$+$

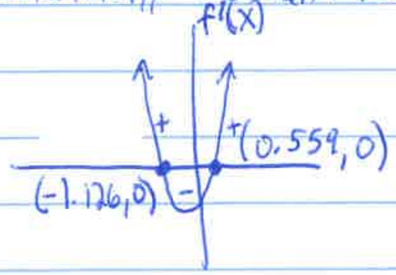
No local extrema! ($k'(x)$ does not change sign)

$k(x)$ is never increasing; $k(x)$ decreases $(-\infty,-2)$, $(-2,2)$, and $(2,\infty)$

27. $f(x) = x^3 - 2x - 2\cos(x)$

$f'(x) = 3x^2 - 2 + 2\sin(x)$

Unfortunately, we can't factor this. Use calculator



CP: $x = -1.126$ and $x = 0.559$

Local max at $x = -1.126$ b/c $f'(x)$ changes from + to − $\rightarrow f(-1.126) = -0.036$

Local min at $x = 0.559$ b/c $f'(x)$ changes from − to + $\rightarrow f(0.559) = -2.639$

$f(x)$ is increasing $(-\infty, -1.126)$ and $(0.559, \infty)$

$f(x)$ is decreasing $(-1.126, 0.559)$

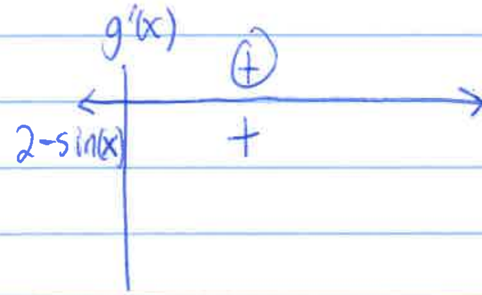
28. $g(x) = 2x + \cos(x)$

$g'(x) = 2 - \sin(x)$

$2 - \sin(x) = 0$

$2 = \sin(x)$

No CP



No local extrema ($g'(x)$ does not change sign)

$g(x)$ increases $(-\infty, \infty)$; $g(x)$ never decreases

53. MVT

$$\frac{f(\pi/3) - f(0)}{\pi/3 - 0} = \frac{\frac{1}{2} - 1}{\pi/3} = \frac{-\frac{1}{2}}{\pi/3} = -\frac{3}{2\pi} \quad \text{(A)}$$

54. $g(x) = e^{x^3 - 6x^2 + 8}$ $\quad g'(x) = e^{x^3-6x^2+8}(3x^2 - 12x) = e^{x^3-6x^2+8}(3x)(x-4)$

$e^{x^3-6x^2+8}$ → always +, $3x$ → $x=0$, $x-4$ → $x=4$

	+ (before 0)	− (0 to 4)	+ (after 4)
$e^{x^3-6x^2+8}$	+	+	+
$3x$	−	+	+
$x-4$	−	−	+

$g(x)$ is decreasing $[0, 4]$ (B)

56. (D) $f(x) = x^{3/5}$

$f'(x) = \frac{3}{5}x^{-2/5} = \frac{3}{5\sqrt[5]{x^2}}$ so $f(x)$ is not differentiable at $x=0$ which is on $[-1, 1]$