Name:
AP Calculus BC

Unit 4 Review Problems

1. The radius of a circle is increasing at a rate of 0.2 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference of the circle is 20π meters?
   a. $0.04\pi\ m^2/sec$
   b. $0.4\pi\ m^2/sec$
   c. $4\pi\ m^2/sec$
   d. $20\pi\ m^2/sec$
   e. $100\pi\ m^2/sec$

$\frac{dr}{dt} = 0.2\ m/s$

$\frac{dA}{dt} = ?$ when $C = 20\pi$

$C = 2\pi r$
$20\pi = 2\pi r$
$r = 10$

$A = \pi r^2$
$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$
$\frac{dA}{dt} = 20\pi(0.2)$
$\frac{dA}{dt} = 4\pi\ m^2/sec$

2. Functions w, x, and y are differentiable with respect to time and are related by the equation $w = x^2 y$. If x is decreasing at a constant rate of 1 unit per minute and y is increasing at a constant rate of 4 units per minute, at what rate is w changing with respect to time when $x = 6$ and $y = 20$?

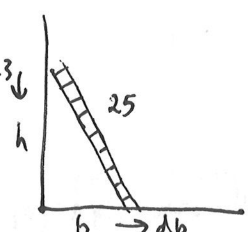
(A) −384 (B) −240 (C) −96 (D) 276 (E) 384

$\frac{dx}{dt} = -1\ u/min$
$\frac{dy}{dt} = 4\ u/min$
$\frac{dw}{dt} = ?$ when $x = 6, y = 20$

$w = x^2 y$ product rule!
$\frac{dw}{dt} = x^2 \frac{dy}{dt} + 2x \frac{dx}{dt} y$
$\frac{dw}{dt} = (6)^2(4) + 2(6)(-1)(20) = 144 - 12(20) = -96$

3. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?
   a. $-\frac{7}{8}\ \frac{ft}{min}$
   b. $\frac{7}{24}\ \frac{ft}{min}$
   c. $-\frac{7}{24}\ \frac{ft}{min}$
   d. $\frac{7}{8}\ \frac{ft}{min}$
   e. $\frac{21}{25}\ \frac{ft}{min}$

$\frac{dh}{dt} = -3$



$\frac{db}{dt} = ?$ when $h = 7$

$b^2 + h^2 = 25^2$
$2b\frac{db}{dt} + 2h\frac{dh}{dt} = 0$
$\frac{db}{dt} = \frac{-2h\frac{dh}{dt}}{2b} \Rightarrow \frac{-h\frac{dh}{dt}}{b} \Rightarrow \frac{-7(-3)}{24}$

what is b?
$b^2 + 7^2 = 25^2$
$b = \sqrt{25^2 - 7^2}$
$b = 24$

$\frac{21}{24} = \boxed{\frac{7}{8}\ ft/min}$

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4. If f'(3)=7 and f(3)=-2, what is the approximation of f(3.1) using the tangent line to the curve of y=f(x) at x=3?

Tangent Line: $y+2=7(x-3)$

Linearization: $y=7(x-3)-2$

at $x=3.1$ $y=7(3.1-3)-2$

$y=7(0.1)-2=-1.3$

$f(3.1)\approx -1.3$

5. If $a(t) = e^t(t-2)(t+3)^3$, then at what t-values is acceleration positive?

Zeroes: $t=2$, $t=-3$
e^t is never 0, always +

sign chart

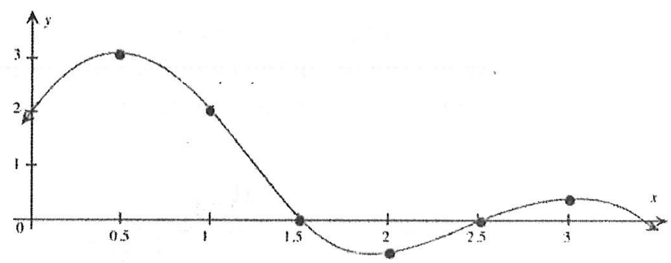
-3 2

	+	−	+
e^t	+	+	+
$(t-2)$	−	−	+
$(t+3)^3$	−	+	+

acceleration is positive on $(-\infty, -3)$ and $(2, \infty)$

6. The graph of v(t) is shown at the right for a particle moving along the
Uh oh! Things got cut off. $0 \le t \le 3.5$

v(t)



a. On what interval(s) is the particle moving right? Justify your answer.

The particle is moving right when $v(t)$ is + (above t-axis): $[0, 1.5)$, $(2.5, 3.5)$

b. On what interval(s) is the particle's speed increasing? Justify your answer.

Speed is increasing when $v(t)$ and $a(t)$ have the same sign: $(0, 0.5)$, $(1.5, 2)$, $(2.5, 3)$

Hint: you can't justify this way, but look for the graph moving away from 0.

c. At what time(s) is the acceleration of the particle 0? Justify your answer.

Acceleration of particle is 0 when slope of $v(t)$ is 0: $t=0.5, 2, 3$

d. At what time(s) does the particle change from moving left to moving right? Justify your answer.

Particle changes from moving left to moving right when $v(t)$ changes from negative to positive: $t=2.5$.

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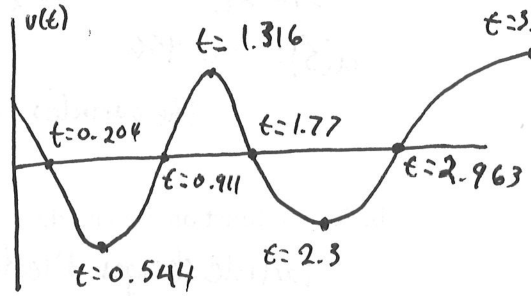
7. CALC OK

a. If $v(t) = 3\sin\left(\frac{1}{2}t^2 - 5t + 1\right)$ models the velocity of a particle for 0<t<4 as the particle moves along the x-axis, on what interval(s) is is the ~~function~~ Particle moving to the right? Justify your answer.

Particle is moving right when $v(t)$ is positive:
$(0, 0.204)$, $(0.911, 1.77)$, $(2.963, 4)$



b. At what time(s) does the particle change direction? Justify your answer.

Particle changes direction when $v(t)$ changes sign:
$t = 0.204, 0.911, 1.77, 2.963$

c. At what time(s) is the acceleration of the particle 0? Justify your answer.

Acceleration of the particle is 0 when the slope of $v(t) = 0$.
$t = 0.544, 1.316, 2.3, 3.996$

d. Is the particle speeding up or slowing down at t=2.5? Justify your answer.

$v(2.5) = -2.602$
$a(2.5) = 3.733$
↳ derivative (use calc)

$v(2.5)$ and $a(2.5)$ have different signs, so the particle is slowing down.

c. What is the velocity of the particle the first time that acceleration is 0? Show the work that leads to your answer.

Acceleration is 0 for the first time at $t = 0.544$. $v(0.544) = -3$

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8. CALC OK

a. For a particle moving along the x-axis, the velocity is given by $v(t) = 2e^{\sin\left(\frac{1}{2}x^2-1\right)} - 2$ for 0<t<5. At t=3, is the speed of the particle increasing or decreasing? Justify.

$v(3) = 2e^{\sin\left(\frac{1}{2}(9)-1\right)} - 2 = -0.592$

$a(3) = -3.956$

The particle's speed is increasing b/c $v(3)$ and $a(3)$ have the same sign

b. For the function above, identify all the times when the particle changes direction. Justify your answer.

Particle changes direction when $v(t)$ changes sign:

$t = 1.414,\ 2.878,\ 3.817,\ 4.566$

9. CALC OK Create a velocity function for a particle moving along the x-axis. The function must satisfy the following properties:

a) The speed of the particle is decreasing at t=2.
b) The acceleration of the particle is is positive at t=2

Answers may vary:

$v(t) = -2t^{-2}$ $\quad a(t) = 4t^{-3}$

$v(2) = -2\left(\frac{1}{4}\right)$ $\quad a(2) = 4\left(\frac{1}{8}\right)$

Speed is decreasing b/c $v(2)$ and $a(2)$ have different signs. $a(2)$ is positive.

10. CALC OK Engineers are testing a new canal for ships. Water is let into the canal at a rate that can be modeled by $W(t) = 42\sin\left(\frac{t}{4} + \cos(t^2)\right)$ where t is hours and W is measured in thousands of gallons. Water drains from the canal at a rate modeled by $D(t) = 42\cos(4t\cos(t))$ where t is hours and D is measured in thousands of gallons.

a. Is the volume of water in the canal increasing or decreasing at t=3.1 hours? Justify your answer.

Overall Rate $= W(t) - D(t)$

$R(t) = W(t) - D(t)$

$R(3.1) = W(3.1) - D(3.1)$

$R(3.1) = -8.669 - 41.343 \Rightarrow$ negative, so the volume of water is decreasing

b. Is the rate at which water is entering the canal increasing or decreasing at t=2.3? Justify your answer.

$W'(2.3) = 74.934$, the rate at which water is entering the canal is increasing b/c $W'(2.3)$ is positive.

Unit 4 Review Problems 1

Name

Calc 1. A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) -0.914

(B) 0.055

(C) 5.486

(D) 6.086

(E) 18.087

$\left.\frac{dv}{dt}\right|_{t=3}$

use calculator

$v'(3) = 0.055$

---

2. A particle moves on the x-axis so that at any time $t, 0 \leq t \leq 1$ its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

$v(t) = 0$

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

(E) 1

$x(t) = \sin(2\pi t) + 2\pi t$

$v(t) = \cos(2\pi t) \cdot 2\pi + 2\pi = 0$

$\cos(2\pi t) \cdot 2\pi = -2\pi$

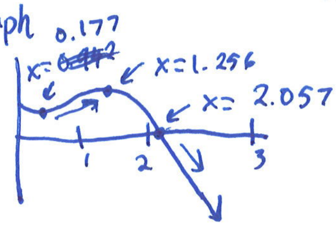
$\cos(2\pi t) = -1$

$2\pi t = \pi$ b/c $\cos(\pi) = -1$

$t = \frac{1}{2}$

Calc 3. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

Graph



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Unit 4 Review Problems 1

(A) $(0,\ 0.177)$ and $(1.256,\ \infty)$

(B) $(0,\ 1.256)$ only

(C) $(0,\ 2.057)$ only

(D) $(0.177,\ 1.256)$ only

(E) $(0.177,\ 1.256)$ and $(2.057,\ \infty)$

Calc

4. A particle moves along a line so that its velocity is given by $v(t) = -t^3 + 2t^2 + 2^{-t}$ for $t \geq 0$. For what values of t is the speed of the particle increasing?

Same problem

(A) $(0,\ 0.177)$ and $(1.256,\ \infty)$

(B) $(0,\ 1.256)$ only

(C) $(0,\ 2.057)$ only

(D) $(0.177,\ 1.256)$ only

(E) $(0.177,\ 1.256)$ and $(2.057,\ \infty)$

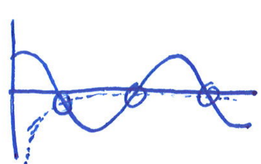
Calc

5. Two particles start at the origin and move along the x-axis. For, $0 \leq t \leq 10$ their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

$v_1 = \cos(t)$

$v_2 = e^{-2t}(-2)$

Graph both. See how many intersections



Three times

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Unit 4 Review Problems 1

(A) None

(B) One

(C) Two

$\boxed{\text{(D) Three}}$

(E) Four

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6. The position of a particle moving along a straight line at any time t is given by $s(t) = t^2 + 4t + 4$. What is the acceleration of the particle when $t = 4$?

(A) 0

$\boxed{\text{(B) 2}}$

(C) 4

(D) 8

(E) 12

$s(t) = t^2 + 4t + 4$
$v(t) = 2t + 4$
$a(t) = 2$

Calc

7. A particle moves along a line so that at time t, where $0 \leq t \leq \pi$, its position is given by $s(t) = -4\cos t - \frac{t^2}{2} + 10$. What is the velocity ofthe particle when its acceleration is zero?

$v(t) = 4\sin(t) - t$

$a(t) = 4\cos(t) - 1 = 0$

Graph a(t) and find 0

$t = 1.3181161$ is when $a(t) = 0$

$\to v(1.3181161) = 2.555$

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(A) -5.19

(B) 0.74

(C) 1.32

(D) 2.55

(E) 8.13

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8. Let x and y be functions of time t such that the sum of x and y is constant. Which of the following equations describes the relationship between the rate of change of x with respect to time and the rate of change of y with respect to time?

(A) $\frac{dx}{dt} = \frac{dy}{dt}$

(B) $\frac{dx}{dt} = -\frac{dy}{dt}$

(C) $\frac{dx}{dt} + \frac{dy}{dt} = \frac{dK}{dt}$, where K is a function of t

(D) $\frac{dx}{dt} + \frac{dy}{dt} = K$, where K is a function of t

$x + y = C$

$\frac{dx}{dt} + \frac{dy}{dt} = 0$

$\frac{dx}{dt} = -\frac{dy}{dt}$

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9. Boyle's law states that if the temperature of an ideal gas is held constant, then the pressure P of the gas and its volume V satisfy the relationship $P = \frac{k}{V}$, where k is a constant. Which of the following best describes the relationship between the rate of change, with respect to time t, of the pressure and the rate of change, with respect to time t, of the volume?

$P = \frac{K}{V}$

$P = KV^{-1}$

$\frac{dP}{dt} = -KV^{-2}\frac{dV}{dt} \rightarrow \frac{dP}{dt} = \frac{-K}{V^2}\frac{dV}{dt}$

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(A) $\frac{dP}{dt} = \frac{k}{\left(\frac{dV}{dt}\right)}$

(B) $\frac{dP}{dt} = \frac{-k}{\left(\frac{dV}{dt}\right)}$

(C) $\frac{dP}{dt} = \frac{k}{V^2}\left(\frac{dV}{dt}\right)$

(D) $\boxed{\frac{dP}{dt} = \frac{-k}{V^2}\left(\frac{dV}{dt}\right)}$

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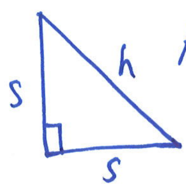
10. An isosceles right triangle with legs of length s has area $A = \frac{1}{2}s^2$. At the instant when $s = \sqrt{32}$ centimeters, the area of the triangle is increasing at a rate of 12 square centimeters per second. At what rate is the length of the hypotenuse of the triangle increasing, in centimeters per second, at that instant?

(A) $\frac{3}{4}$

(B) $\boxed{3}$

(C) $\sqrt{32}$

(D) 48



$A = \frac{1}{2}s^2$

$\frac{dA}{dt} = 12\ cm^2/s$ when $s = \sqrt{32}$

$\frac{dh}{dt} = ?$

$s^2 + s^2 = h^2$

$2s^2 = h^2$

$4s\frac{ds}{dt} = 2h\frac{dh}{dt}$

Help: $2\sqrt{32}^2 = h^2$

$64 = h^2$

$8 = h$

$A = \frac{1}{2}s^2$

$\frac{dA}{dt} = s\frac{ds}{dt}$

$12 = \sqrt{32}\frac{ds}{dt}$

$\frac{12}{\sqrt{32}} = \frac{ds}{dt}$

$\frac{dh}{dt} = \frac{4s\frac{ds}{dt}}{2h} = \frac{4\sqrt{32}\left(\frac{12}{\sqrt{32}}\right)}{2 \cdot 8} = \frac{48}{16} = \boxed{3}$

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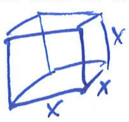
11. A cube with edges of length x centimeters has volume $V(x) = x^3$ cubic centimeters. The volume is increasing at a constant rate of 40 cubic centimeters per minute. At the instant when $x = 2$, what is the rate of change of x, in centimeters per minute, with respect to time?



$\frac{dV}{dt} = 40\ cm^3/min$

when $x = 2$, $\frac{dx}{dt} = ?$

$V = x^3$

$\frac{dV}{dt} = 3x^2\frac{dx}{dt}$

$40 = 3(2)^2\frac{dx}{dt}$

$40 = 12\frac{dx}{dt}$

$\frac{40}{12} = \frac{dx}{dt}$ $\quad \frac{40}{12} = \frac{10}{3}$

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$\boxed{\text{(A) } 10/3}$

(B) $\sqrt{\frac{40}{3}}$

(C) 5

(D) 10

12. The function f is twice differentiable with $f(2) = 1$, $f'(2) = 4$, and $f''(2) = 3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of f at $x = 2$?

(A) 0.4

$\boxed{\text{(B) } 0.6}$

(C) 0.7

(D) 1.3

(E) 1.4

$y-1=4(x-2)$

$y=4(x-2)+1$

$y=4(1.9-2)+1$

$y=4(-0.1)+1$

$y=0.6$

13. For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$?

$f'(1)=2+1=3$

$y-4=3(x-1)$

$y=3(x-1)+4$

$y=3(1.2-1)+4$

$y=3(0.2)+4$

$y=4.6$

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Unit 4 Review Problems 1

(A) 0.6

(B) 3.4

(C) 4.2

(D) 4.6

(E) 4.64

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14. Let f be the function given by $f(x) = 2\cos x + 1$. What is the approximation for $f(1.5)$ found by using the line tangent to the graph of f at $x = \pi/2$?

(A) -2

(B) 1

(C) $\pi - 2$

(D) $4 - \pi$

Point: $f\left(\frac{\pi}{2}\right) = 2\cos\left(\frac{\pi}{2}\right) + 1 = 1$

$\left(\frac{\pi}{2}, 1\right)$

slope: $f'(x) = -2\sin(x)$

$f'\left(\frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}\right) = -2$

$y - 1 = -2\left(x - \frac{\pi}{2}\right)$

$y = -2\left(x - \frac{\pi}{2}\right) + 1 \rightarrow y = -2x + \pi + 1$

when $x = 1.5$

$y = -2(1.5) + \pi + 1$

$y = -3 + \pi + 1 = -2 + \pi$

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