Practice:

2. Air is being pumped into a spherical balloon at the rate of 7 cubic centimeters per second. What is the rate of change of the radius at the instant the volume equals 36π? The volume of a sphere of radius r is $\frac{4\pi}{3}r^3$.

$V = \frac{4\pi}{3} r^3$

$36\pi = \frac{4\pi}{3} r^3$

$\frac{dr}{dt} = ?$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{3}{4\pi} \cdot 36\pi = r^3$

$\frac{dV}{dt} = 7 \text{ cm}^3/\text{s}$

$27 = r^3$

$r = 3$ cm

$\boxed{r = 3}$

$7 = 4\pi(3)^2 \frac{dr}{dt}$

$7 = 36\pi \frac{dr}{dt}$

$\frac{7}{36\pi} = \frac{dr}{dt}$ $\quad \frac{7}{36\pi}$ cm/s

Solve each related rate problem.

1) Water leaking onto a floor forms a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm?

$\frac{dr}{dt} = 4$ cm/min $\quad \frac{dA}{dt} = ? \quad r = 5$

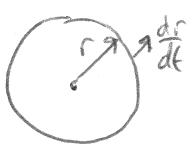
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(5)(4) = \boxed{40\pi \text{ cm}^2/\text{min}}$

2) Oil spilling from a ruptured tanker spreads in a circle on the surface of the ocean. The area of the spill increases at a rate of 9π m²/min. How fast is the radius of the spill increasing when the radius is 10 m?

rate



$\frac{dA}{dt} = 9\pi \text{ m}^2/\text{min} \quad \frac{dr}{dt} = ? \quad r = 10\text{m}$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

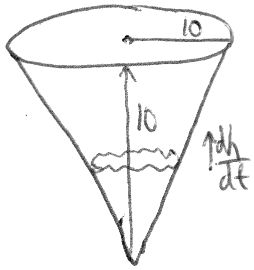
$9\pi = 2\pi(10) \frac{dr}{dt}$

$\frac{9}{20} = \frac{dr}{dt}$

$\boxed{\frac{9}{20} \text{ m/min}}$

3) A conical paper cup is 10 cm tall with a radius of 10 cm. The cup is being filled with water so that the water level rises at a rate of 2 cm/sec. At what rate is water being poured into the cup when the water level is 8 cm?

$\hookrightarrow \frac{dh}{dt}$ $\hookrightarrow \frac{dV}{dt}$



$\frac{dh}{dt} = 2\text{cm/s}$

$\frac{dV}{dt} = ?$

$h = 8$

similar triangles

$\frac{r}{h} = 1$ so if $h = 8$, $r = 8$

For this cup: $\rightarrow r = h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi h^2 h$

$V = \frac{1}{3}\pi h^3$

$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$

$\frac{dV}{dt} = \pi(8)^2(2) = 128\pi \text{ cm}^3/\text{s}$

4) A spherical balloon is inflated so that its radius (r) increases at a rate of $\frac{2}{r}$ cm/sec. How fast is the volume of the balloon increasing when the radius is 4 cm?

$\hookrightarrow \frac{dV}{dt} = ?$ $r = 4$ $\frac{dr}{dt} = \frac{2}{r}$

$V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(4)^2\left(\frac{2}{4}\right) = 32\pi \text{ cm}^3/\text{s}$

Watch the video online. Take notes.