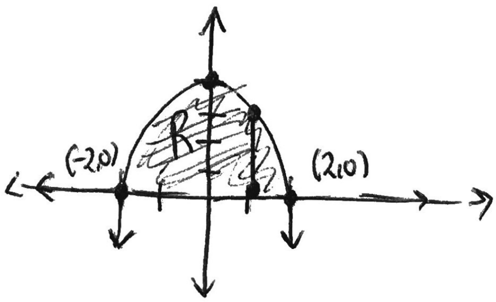
HW 8.9 page 2

1. $y = 4 - x^2 \quad y = 0$



a) Vertical!

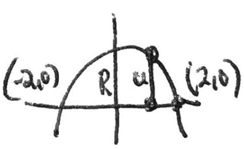
$A = \int_{-2}^{2} \text{upper} - \text{lower}\, dx$

$A = \int_{-2}^{2} (4 - x^2) - 0\, dx = \int_{-2}^{2} 4 - x^2 dx = \left(4x - \frac{1}{3}x^3\right)\Big|_{-2}^{2}$

$= \left(8 - \frac{1}{3}(2)^3\right) - \left(-8 - \frac{1}{3}(-2)^3\right) = 8 - \frac{8}{3} - \left(-8 + \frac{8}{3}\right)$

$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{48}{3} - \frac{16}{3} = \frac{32}{3}$

b. Perp to x-axis: vertical



Square: $\square$ u

$A = u^2$

$u = (4 - x^2) - 0$

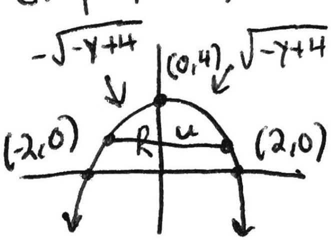
$u = (4 - x^2)$

$A = (4 - x^2)^2 \quad V = \int_{-2}^{2} (4 - x^2)^2 dx = \int_{-2}^{2} 16 - 8x^2 + x^4 dx$

use symmetry: $2 \int_{0}^{2} 16 - 8x^2 + x^4 dx$

$2\left(16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right)\Big|_0^2 = \boxed{2\left(16(2) - \frac{8}{3}(2)^3 + \frac{1}{5}(2)^5\right)}$

c. perp to y-axis: horizontal



$\square$ u

everything in y!

$A = u^2$

$u = \sqrt{-y+4} - (-\sqrt{-y+4})$

$u = \sqrt{-y+4} + \sqrt{-y+4}$

$u = 2\sqrt{-y+4}$

$A = (2\sqrt{-y+4})^2$

$A = 4(-y+4)$

$A = -4y + 16$

$V = \int_{0}^{4} -4y + 16\, dy$

$V = (-2y^2 + 16y)\Big|_0^4$

$V = -2(4)^2 + 16(4)$

$V = -32 + 64 = \boxed{32}$

$y = 4 - x^2$

$y - 4 = -x^2$

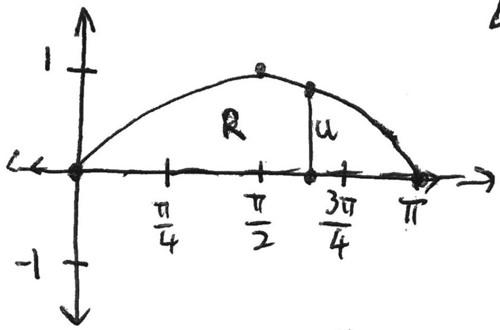
$-y + 4 = x$

$\pm\sqrt{-y+4} = x$

Not the same shape b/c direction of cross sections is different.

Not the same volume

2. a)



$A = \frac{\sqrt{3}}{4} u^2$ $\quad$ $A = \frac{\sqrt{3}}{4} \sin^2(x)$

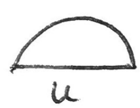
$u = $ upper $-$ lower

$u = \sin(x) - 0$ $\quad$ $V = \int_0^{\pi} \frac{\sqrt{3}}{4} \sin^2(x)\, dx$

$u = \sin(x)$ $\quad$ Need calc for this, no anti-deriv rule

$\boxed{V = 0.680}$

b) vertical again



$A = \frac{\pi u^2}{8}$

$u = \sin(x) - 0$

$u = \sin(x)$

$A = \frac{\pi \sin^2(x)}{8}$

$V = \int_0^{\pi} \frac{\pi \sin^2(x)}{8}\, dx$

Need calc again.

$\boxed{V = 0.617}$