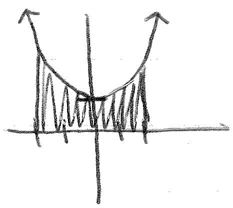
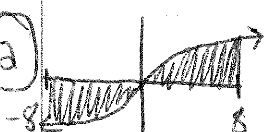
Areas in the Plane Hwk

① 

$$A=\int_{-2}^{2}(x^2+1)\,dx = 2\int_{0}^{2}(x^2+1)\,dx = 2\left[\frac{x^3}{3}+x\right]_0^2 =$$

$$2\left[\left(\frac{8}{3}+2\right)-0\right] = \frac{16}{3}+4 = \boxed{\frac{28}{3}}$$

② 

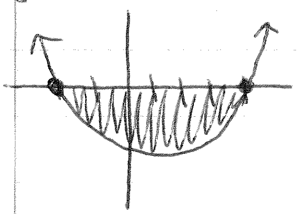
$$A=\int_{-8}^{0}\left|\sqrt[3]{x}\right|dx + \int_{0}^{8}\sqrt[3]{x}\,dx = 2\int_{0}^{8}\sqrt[3]{x}\,dx = 2\int_{0}^{8}x^{1/3}\,dx$$

$$=2\cdot\frac{3x^{4/3}}{4}\Big]_0^8 = \frac{3}{2}\left[8^{4/3}-0\right] = \frac{3}{2}\left(2^4\right) = \boxed{24}$$

③ $y=x^2-2x-8$

$y=(x-4)(x+2)$

$y=0$ at $x=4,\ x=-2$

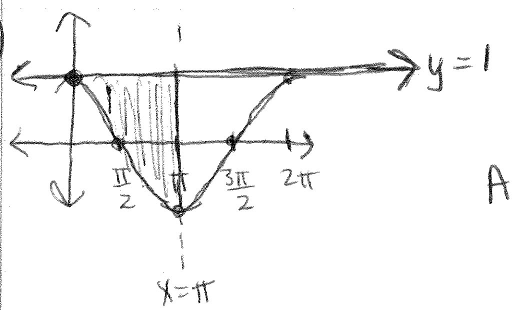


$$A=-\int_{-2}^{4}(x^2-2x-8)\,dx = -\left(\frac{x^3}{3}-\frac{2x^2}{2}-8x\right]_{-2}^{4} =$$

$$-\left[\left(\frac{4^3}{3}-4^2-8(4)\right)-\left(\frac{-8}{3}-4+16\right)\right]$$

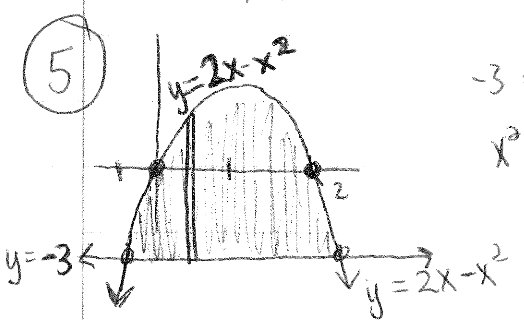
$$-\left[\frac{64}{3}-16-32+\frac{8}{3}+4-16\right]$$

$$-\left[\frac{72}{3}-64+4\right] = -\left[24-64+4\right] = -\left[-36\right] = \boxed{36}$$

④ 

$$A=\int_{0}^{\pi}(1-\cos x)\,dx = x-\sin x\Big]_0^{\pi}$$

$$A=(\pi-\underbrace{\sin\pi}_{0})-(0-\sin 0) = \boxed{\pi}$$

⑤ 

$-3=2x-x^2$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

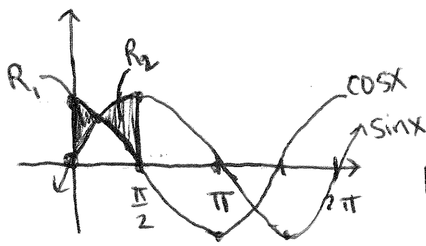
$x=3,\ x=-1$

$$A=\int_{-1}^{3}\left[(2x-x^2)-(-3)\right]dx = \int_{-1}^{3}(2x-x^2+3)\,dx$$

$$A=\frac{2x^2}{2}-\frac{x^3}{3}+3x\Big]_{-1}^{3} = \left(9-\frac{27}{3}+9\right)-\left(1+\frac{1}{3}-3\right)$$

$$A=9-1+3-\frac{1}{3} = \boxed{\frac{32}{3}}$$

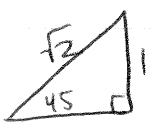
(6)



$\cos x = \sin x$ when $x = \pi/4$

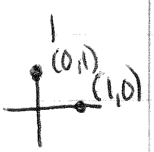
$$A = \int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$A = \sin x + \cos x \Big]_0^{\pi/4} + -\cos x - \sin x \Big]_{\pi/4}^{\pi/2} = (\sin(\pi/4) + \cos(\pi/4)) - (\sin 0 + \cos 0) + -\cos(\pi/2) - \sin(\pi/2) - (-\cos(\pi/4) - \sin(\pi/4))$$



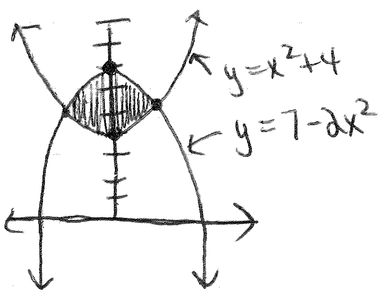
$$A = \boxed{\sin(\pi/4)} + \cos(\pi/4) - \sin 0 - \cos 0 - \cos(\pi/2) - \sin(\pi/2) + \cos(\pi/4) + \boxed{\sin(\pi/4)}$$

$$A = 2\sin(\pi/4) + 2\cos(\pi/4) - \sin 0 - \cos 0 - \cos(\pi/2) - \sin(\pi/2)$$



$$A = 2\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{\sqrt{2}}\right) - 0 - 1 - 0 - 1 = \boxed{4\left(\frac{1}{\sqrt{2}}\right) - 2}$$

(7)



$$A = \int_{-1}^{1} [(7 - 2x^2) - (x^2 + 4)]\,dx$$

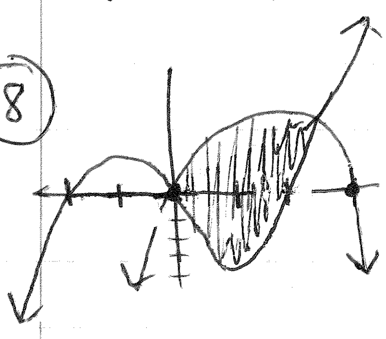
$x^2 + 4 = 7 - 2x^2$

$3x^2 = 3$

$x = \pm 1$

$$A = 2\int_0^1 (3 - 3x^2)\,dx = 3x - \cancel{\frac{3x^3}{3}}\Big]_0^1 = 2(3 - 1) + 0 = 2(2) = \boxed{4}$$

(8)



2 -1 -5

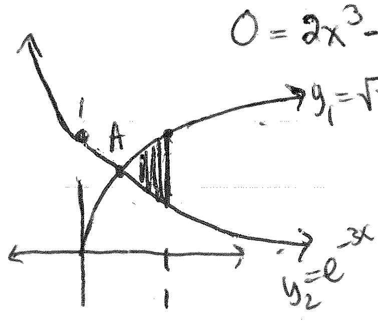
$y_1 = -x^2 + 3x \quad y_2 = 2x^3 - x^2 - 5x$

$$\int_0^2 [(-x^2 + 3x) - (2x^3 - x^2 - 5x)]\,dx = \boxed{8}$$

$-x^2 + 3x = 2x^3 - x^2 - 5x$

$0 = 2x^3 - 8x = 2x(x^2 - 4)$

(9)



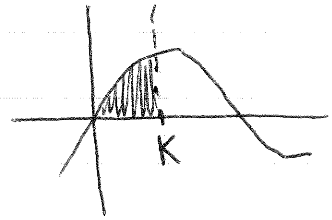
$y_1 = y_2$

$x = .238734$

STO → A

$$\int_A^1 (\sqrt{x} - e^{-3x})\,dx = \boxed{.443}$$

(10)



$$\int_0^K \sin(2x)\,dx = 0.1$$

$$-\frac{1}{2}\cos(2x)\Big]_0^K = -\frac{1}{2}\cos(2k) + \frac{1}{2}\cos(0) = 0.1$$

$$-\frac{1}{2}\cos(2k) = 0.1 - 0.5 = -0.4$$

$$\cos(2k) = -\frac{4}{10}\left(\frac{-2}{1}\right)$$

$$\cos(2k) = \frac{8}{10}$$

graph $y_1 = \cos(2k)$
and $y_2 = 8/10$

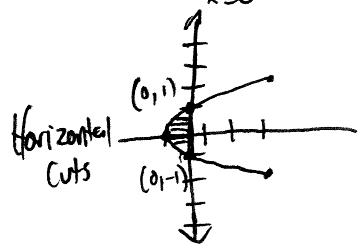
$\boxed{\text{intersect at } k = .32175}$

8.8 Areas Between Curves Day 2

The area of the region enclosed by the graph of $x = y^2 - 1$ and the y-axis is

A. -4/3 B. 0 C. 2/3 D. 4/3 E. 8/3

x=0 is the y-axis



x	y
3	-2
0	-1
-1	0
0	1
3	2

Horiz: $\int$ right-left dy = $\int_{-1}^{1} 0 - (y^2 - 1)\,dy$

$\int_{-1}^{1} -y^2 + 1\,dy = -\frac{1}{3}y^3 + y\Big|_{-1}^{1} = \left(\frac{-1}{3}+1\right) - \left(\frac{1}{3}-1\right)$

$= -\frac{1}{3} - \frac{1}{3} + 1 + 1 = 2 - \frac{2}{3}$

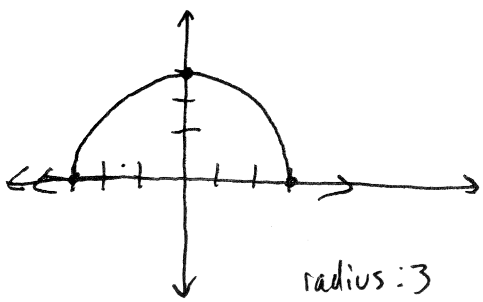
$= \frac{4}{3}$

The area of the region enclosed by the graph of $y = \sqrt{9 - x^2}$ and the x-axis is

semi-circle

(A) 36 (B) $\frac{9\pi}{2}$ (C) 9π

(D) 18π (E) 36π



x	y
-3	0
-2	$\sqrt{5}$
-1	$\sqrt{8}$
0	3
1	$\sqrt{8}$
2	$\sqrt{5}$
3	0

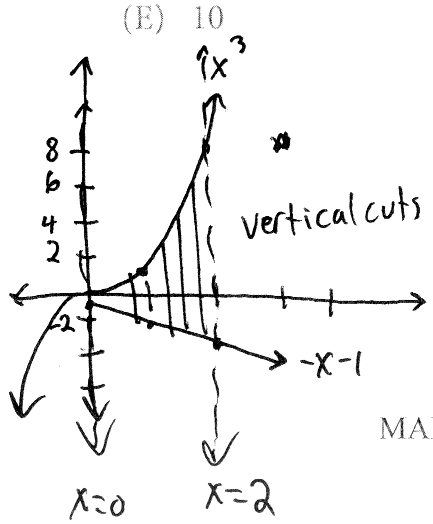
Don't need $\int$: the shape is a semi-circle.

radius: 3 Area = $\frac{\pi(3)^2}{2} = \frac{9\pi}{2}$

What is the area of the region between the graphs of $y = x^3$ and $y = -x - 1$ from $x = 0$ to $x = 2$?

(A) 0 (B) 4 (C) 5

(D) 8 (E) 10



$y = x^3$

x	y
0	0
1	1
2	8

vertical OK

$\int$ upper-lower dx

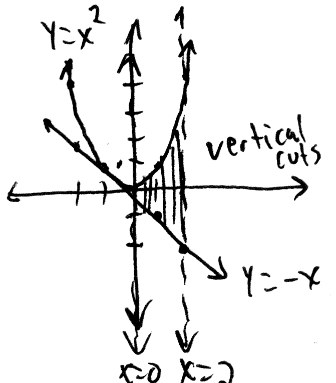
$\int_0^2 x^3 - (-x-1)\,dx = \int_0^2 x^3 + x + 1\,dx$

$\frac{1}{4}x^4 + \frac{1}{2}x^2 + x\Big|_0^2 = \frac{1}{4}(16) + \frac{1}{2}(4) + 2$

$= 4 + 2 + 2 = 8$

What is the area of the region between the graphs of $y = x^2$ and $y = -x$ from $x = 0$ to $x = 2$?

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$ (C) 4 (D) $\frac{14}{3}$ (E) $\frac{16}{3}$



$y=x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

$y=-x$

x	y
-2	2
-1	1
0	0
1	-1
2	-2

$\int$ upper−lower dx

$\int_0^2 x^2-(-x)\,dx$

$\int_0^2 x^2+x\,dx = \frac{1}{3}x^3+\frac{1}{2}x^2 \Big|_0^2$

$= \frac{1}{3}(8)+\frac{1}{2}(4)$

$= \frac{8}{3}+2 = \frac{8}{3}+\frac{6}{3} = \frac{14}{3}$

Find the area of the region bounded by the line $y = 2x$ and the parabola $y = x^2 - 4x$.

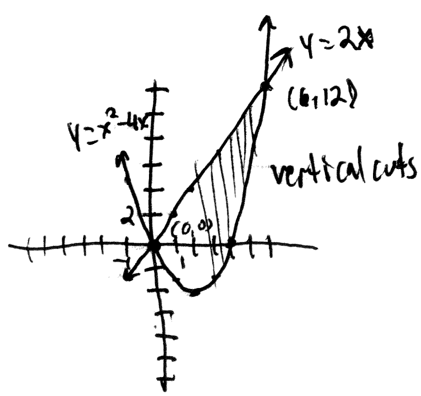
(a) 63

(b) 36

(c) 25

(d) 52

(e) 48



$y=2x$

x	y
-2	-4
-1	-2
0	0
1	2
2	4
3	6
4	8
5	10
6	12

$y=x^2-4x$

x	y
-2	12
-1	5
0	0
1	-3
2	-4
3	-3
4	0
5	5
6	12

$\int_0^6 2x-(x^2-4x)\,dx = \int_0^6 2x-x^2+4x\,dx = \int_0^6 -x^2+6x\,dx$

$-\frac{1}{3}x^3+3x^2 \Big|_0^6 = -\frac{1}{3}(6)^3+3(6)^2 = -72+3\cdot 6^2 = 36$

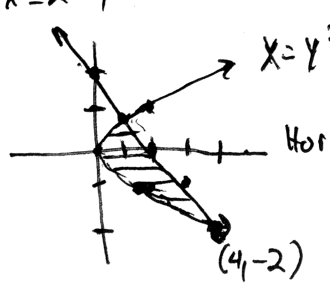
8. Determine the area enclosed by the parabola $x = y^2$ and the line x+y=2.

a. 5/2 b. 3/2 c. 11/6 d. 9/2 e. 29/6

$x = 2-y$

$x=y^2$

x	y
4	-2
1	-1
0	0
1	1
4	2



x	y
4	-2
3	-1
2	0
1	1
0	2

$\int_{-2}^1 (2-y)-(y^2)\,dy$

$\int_{-2}^1 2-y-y^2\,dy = 2y-\frac{1}{2}y^2-\frac{1}{3}y^3 \Big|_{-2}^1$

$(2-\frac{1}{2}-\frac{1}{3})-(-4-2+\frac{8}{3}) = 2-\frac{1}{2}-\frac{1}{3}+4+2-\frac{8}{3}$

4.5 or $\frac{9}{2}$