6.5 PS Answers

Evaluate the following trigonometric expressions.

a. $2\sin\left(\frac{\pi}{8}\right)\cos\left(\frac{\pi}{8}\right)$ going backwards using $\sin(2\theta)$

$\theta = \frac{\pi}{8}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

b. $\frac{1}{2}\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) = \frac{1}{4}\left(2\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right)\right)$ use $\sin(2\theta)$

$\sin\left(2\cdot\frac{\pi}{12}\right)$ $\theta = \frac{\pi}{12}$

$\frac{1}{4}\left(\sin\left(\frac{\pi}{6}\right)\right) = \frac{1}{8}$

c. $4\sin\left(-\frac{5\pi}{12}\right)\cos\left(-\frac{5\pi}{12}\right) = 2\left(2\sin\left(\frac{-5\pi}{12}\right)\cos\left(\frac{-5\pi}{12}\right)\right)$

$2\sin\left(\frac{-5\pi}{6}\right) = -2\sin\left(\frac{\pi}{6}\right) = -2\left(\frac{1}{2}\right) = -1$ $\theta = \frac{-5\pi}{12}$

d. $\cos^2\left(\frac{3\pi}{8}\right) - \sin^2\left(\frac{3\pi}{8}\right)$ $\cos(2\theta)$

$\theta = \frac{3\pi}{8}$

$\cos\left(\frac{3\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

e. $2\cos^2\left(\frac{\pi}{12}\right) - 1$ $\cos(2\theta)$ check lesson Summary identities

$\theta = \frac{\pi}{12}$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

f. $1 - 2\sin^2\left(-\frac{\pi}{8}\right)$ $\cos(2\theta)$ check lesson summary identities

$\theta = \frac{-\pi}{8}$

$\cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

h. $\dfrac{2\tan\left(\frac{\pi}{8}\right)}{1-\tan^2\left(\frac{\pi}{8}\right)}$ $\tan(2\theta)$ $\theta = \frac{\pi}{8}$

$\tan\left(\frac{\pi}{4}\right) = 1$

i. $\dfrac{2\tan\left(-\frac{5\pi}{12}\right)}{1-\tan^2\left(-\frac{5\pi}{12}\right)}$ $\tan(2\theta)$ $\theta = -\frac{5\pi}{12}$

$\tan\left(-\frac{5\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

j. $\cos^2\left(\frac{\pi}{8}\right)$ $\left[\cos\left(\frac{\pi}{8}\right)\right]^2 = \left(\sqrt{\frac{1+\cos\left(\frac{\pi}{4}\right)}{2}}\right)^2$ $\cos\left(\frac{\pi}{8}\right) = \cos\left(\frac{\frac{\pi}{4}}{2}\right)$ $\theta = \frac{\pi}{4}$

$\cos^2\left(\frac{\pi}{8}\right) = \dfrac{1+\cos\left(\frac{\pi}{4}\right)}{2} = \dfrac{1+\frac{\sqrt{2}}{2}}{2} = \dfrac{1}{2}+\dfrac{\sqrt{2}}{4}$

k. $\cos\left(\frac{\pi}{8}\right)$

Rotation by $\theta = \frac{\pi}{8}$ terminates in Quadrant I; therefore, $\cos\left(\frac{\pi}{8}\right)$ has a positive value.

$\cos\left(\frac{\pi}{8}\right) = \sqrt{\dfrac{1+\cos\left(\frac{\pi}{4}\right)}{2}} = \sqrt{\dfrac{1}{2}+\dfrac{\sqrt{2}}{4}} = \dfrac{\sqrt{2+\sqrt{2}}}{2}$

$\cos\left(\frac{\frac{\pi}{4}}{2}\right)$

l. $\cos\left(-\frac{9\pi}{8}\right)$

Rotation by $\theta = -\frac{9\pi}{8}$ terminates in Quadrant II; therefore, $\cos\left(-\frac{9\pi}{8}\right)$ has a negative value.

$\cos\left(-\frac{9\pi}{8}\right) = -\sqrt{\dfrac{1+\cos\left(-\frac{9\pi}{4}\right)}{2}} = -\sqrt{\dfrac{1+\cos\left(\frac{\pi}{4}\right)}{2}} = -\sqrt{\dfrac{1}{2}+\dfrac{\sqrt{2}}{4}} = -\dfrac{\sqrt{2+\sqrt{2}}}{2}$

$\cos\left(\frac{-\frac{9\pi}{4}}{2}\right)$