

Problem Set

Example 1: Find the exact value of $\cos 15^\circ$.

$$\cos(15) = \cos(45 - 30) = \cos(45)\cos(30) + \sin(45)\sin(30)$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

Example 2: Find the exact value of $\sin \frac{13\pi}{12}$.

$$\sin\left(\frac{13\pi}{12}\right) = \sin\left(\frac{8\pi}{12} + \frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{5\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{5\pi}{6}\right)$$

$$\frac{\sqrt{2}}{2} \left(-\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{2}}{2} \left(\frac{1}{2}\right) = \frac{-\sqrt{6} + \sqrt{2}}{4}$$

Example 3: Find the exact value of $\tan \frac{17\pi}{12}$.

$$\tan\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right) = \tan\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$$

$$= \frac{\tan\left(\frac{3\pi}{4}\right) + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan\left(\frac{3\pi}{4}\right)\tan\left(\frac{2\pi}{3}\right)} = \frac{-1 + (-\sqrt{3})}{1 - (-1)(-\sqrt{3})} = \frac{-1 - \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{(1 + \sqrt{3})}{(1 + \sqrt{3})}$$

$$\text{If you simplify: } \frac{-1 - \sqrt{3} - \sqrt{3} - 3}{1 - 3} = \frac{-4 - 2\sqrt{3}}{-2} = \boxed{2 + \sqrt{3}}$$

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Example 5: Write each as a single trigonometric function:

a. $\cos 4x \cos 3x + \sin 4x \sin 3x = \cos(4x - 3x) = \cos(x)$

b. $\sin 2x \cos x - \sin x \cos 2x = \sin(2x - x) = \sin(x)$

c. $\frac{\tan 5x - \tan 4x}{1 + \tan 5x \tan 4x} = \tan(5x - 4x) = \tan(x)$

d. $\sin 10^\circ \cos 5^\circ + \cos 10^\circ \sin 5^\circ = \sin(10 + 5) = \sin(15^\circ)$

e. $\cos 37^\circ \cos 22^\circ - \sin 37^\circ \sin 22^\circ = \cos(37 + 22) = \cos(59^\circ)$