Lesson Summary

For all real numbers θ for which the expressions are defined,

$\sin(\theta) = \sin(2\pi n + \theta)$ and $\cos(\theta) = \cos(2\pi n + \theta)$ for all integer values of n

$\tan(\theta) = \tan(\pi n + \theta)$ for all integer values of n

$\sin(-\theta) = -\sin(\theta)$, $\cos(-\theta) = \cos(\theta)$, and $\tan(-\theta) = -\tan(\theta)$

$\sin\left(\frac{\pi}{2} + \theta\right) = \cos(\theta)$ and $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin(\theta)$

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$ and $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$

Problem Set Use the identities. Sample work shown for some problems

1. Evaluate the following trigonometric expressions. Show how you used the unit circle to determine the solution.

a. $\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{12\pi}{6} + \frac{\pi}{6}\right) = \sin(2\pi + \frac{\pi}{6}) = \sin(\frac{\pi}{6}) = \frac{1}{2}$

b. $\cos\left(-\frac{5\pi}{3}\right) = \cos(\frac{5\pi}{3}) = \cos\left(\frac{6\pi}{3} - \frac{\pi}{3}\right) = \cos(2\pi - \frac{\pi}{3}) = \cos(-\frac{\pi}{3}) = \cos(\frac{\pi}{3}) = \frac{1}{2}$

c. $\tan\left(\frac{25\pi}{4}\right) = 1$

d. $\sin\left(-\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

e. $\cos\left(-\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

f. $\sin\left(\frac{17\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

g. $\cos\left(\frac{25\pi}{4}\right) = \cos\left(\frac{24\pi}{4} + \frac{\pi}{4}\right) = \cos\left(6\pi + \frac{\pi}{4}\right) = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

h. $\tan\left(\frac{29\pi}{6}\right) = \tan\left(\frac{24\pi}{6} + \frac{5\pi}{6}\right) = \tan(4\pi + \frac{5\pi}{6}) = \tan(\frac{5\pi}{6}) = \tan(\pi - \frac{\pi}{6}) = \tan(-\frac{\pi}{6})$
$= -\tan(\frac{\pi}{6}) = -\frac{\sqrt{3}}{3}$

i. $\sin\left(-\frac{31\pi}{6}\right) = \frac{1}{2}$

j. $\cos\left(-\frac{32\pi}{6}\right) = -\frac{1}{2}$

k. $\tan\left(-\frac{18\pi}{3}\right) = 0$

2. Given each value of β below, find two values of α with $0 \le \alpha \le 2\pi$ so that $\sin(\alpha) = \cos(\beta)$.

d. $\beta = \frac{\pi}{3}$ $\cos(\frac{\pi}{3}) = \frac{1}{2}$ $\sin(\alpha) = \frac{1}{2}$ α must be either $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

e. $\beta = \frac{5\pi}{6}$ $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$ $\sin(\alpha) = -\frac{\sqrt{3}}{2}$ $\alpha = \frac{4\pi}{3}, \frac{5\pi}{3}$

f. $\beta = \frac{7\pi}{4}$ $\cos(\frac{7\pi}{4}) = \frac{\sqrt{2}}{2}$ $\sin(\alpha) = \frac{\sqrt{2}}{2}$ $\alpha = \frac{\pi}{4}, \frac{3\pi}{4}$

g. $\beta = \frac{\pi}{12}$ $\cos(\frac{\pi}{12})$ $\alpha = \frac{5\pi}{12}, \frac{7\pi}{12}$

3. Jamal thinks that $\cos\left(\alpha - \frac{\pi}{4}\right) = \sin\left(\alpha + \frac{\pi}{4}\right)$ for any value of α. Is he correct? Explain how you know.

Jamal is correct; If $\theta = \alpha + \frac{\pi}{4}$ $\theta - \frac{\pi}{2} = \alpha + \frac{\pi}{4} - \frac{\pi}{2} = \alpha - \frac{\pi}{4}$

Since $\cos(\theta - \frac{\pi}{2}) = \sin(\theta)$

$\cos(\alpha - \frac{\pi}{4}) = \sin(\alpha + \frac{\pi}{4})$

4. Shawna thinks that $\cos\left(\alpha - \frac{\pi}{3}\right) = \sin\left(\alpha + \frac{\pi}{6}\right)$ for any value of α. Is she correct? Explain how you know.

Shawna is correct; reasoning is the same.

Example answers

5. A frog is sitting on the edge of a playground carousel with radius 1 meter. The ray through the frog's position and the center of the carousel makes an angle of measure θ with the horizontal, and his starting coordinates are approximately $(0.81, 0.59)$. Find his new coordinates after the carousel rotates by each of the following amounts.

h. $\frac{\pi}{2}$ h. $\cos(\theta + \frac{\pi}{2}) = -\sin(\theta) = -0.59$ $(-0.59, 0.81)$
$\sin(\theta + \frac{\pi}{2}) = \cos(\theta) = 0.81$

i. π

j. 2π j. $\cos(\theta + 2\pi) = \cos(\theta) = 0.81$ $(0.81, 0.59)$
$\sin(\theta + 2\pi) = \sin(\theta) = 0.59$

k. $-\frac{\pi}{2}$

l. $-\pi$ l. $\cos(\theta - \pi) = -\cos(\theta) = -0.81$
$\sin(\theta - \pi) = -\sin(\theta) = -0.59$

m. $\frac{\pi}{2} - \theta$

n. $\pi - 2\theta$

o. -2θ