Problem Set

4. Identify the quadrant of the plane that contains the terminal ray of a rotation by θ if θ satisfies the given conditions.

   a. $\sin(\theta) > 0$ and $\cos(\theta) > 0$ Quadrant: 1

   b. $\sin(\theta) < 0$ and $\cos(\theta) < 0$ Quadrant: 3

   c. $\sin(\theta) < 0$ and $\tan(\theta) > 0$ Quadrant: 3

   d. $\tan(\theta) > 0$ and $\sin(\theta) > 0$ Quadrant: 1

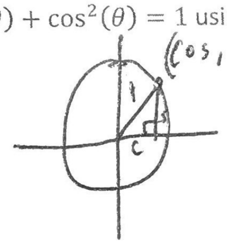
   e. $\tan(\theta) < 0$ and $\sin(\theta) > 0$ Quadrant: 2

   f. $\tan(\theta) < 0$ and $\cos(\theta) > 0$ Quadrant: 4

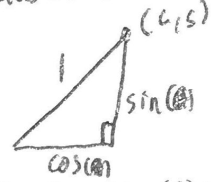
   g. $\cos(\theta) < 0$ and $\tan(\theta) > 0$ Quadrant: 3

   h. $\sin(\theta) > 0$ and $\cos(\theta) < 0$ Quadrant: 2

5. Explain why $\sin^2(\theta) + \cos^2(\theta) = 1$ using the unit circle.

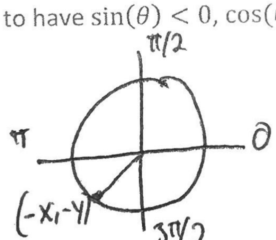


Radius of UC is 1. That is the hypotenuse of a right triangle w/ $x = \cos(\theta)$ $y = \sin(\theta)$



Pythag: $x^2 + y^2 = 1$ OR $\sqrt{\cos^2(\theta) + \sin^2(\theta)} = 1$

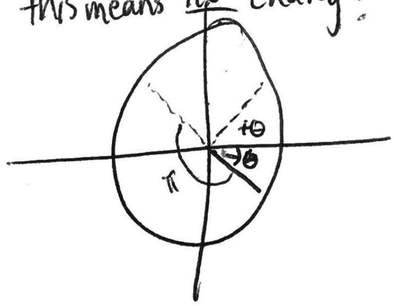
6. Explain how it is possible to have $\sin(\theta) < 0$, $\cos(\theta) < 0$, and $\tan(\theta) > 0$. For which values of θ between 0 and 2π does this happen?



If sine and cosine are both negative, but tangent is positive, that puts θ in quadrant 3. $\tan(\theta) = \frac{-y}{-x} = \frac{y}{x}$. $\boxed{\pi < \theta < \frac{3\pi}{2}}$

7. Duncan says that for any real number θ, $\tan(\theta) = \tan(\pi - \theta)$. Is he correct? Explain how you know.

False! $\tan(\pi - \theta)$ is half a circle from $-\theta$: for tan, this means no change: $\tan(\pi - \theta) = \tan(-\theta)$

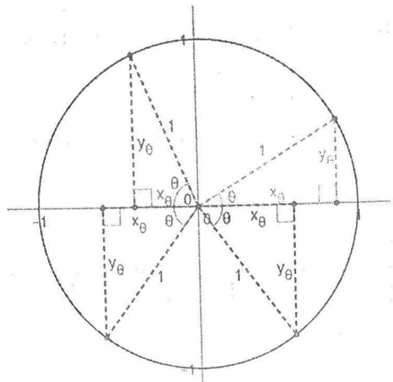


But this does not = $\tan(\theta)$...

$\tan(-\theta) = -\tan(\theta)$

Check our identities.

8. Given the following trigonometric functions, identify the quadrant in which the terminal ray of θ lies in the unit circle shown below. Find the other two trigonometric functions of θ of $\sin(\theta)$, $\cos(\theta)$, and $\tan(\theta)$.



a. $\sin(\theta) = \frac{1}{2}$ and $\cos(\theta) > 0$. ; 1st Quad: $\cos(\theta) = \frac{\sqrt{3}}{2}$ $\tan(\theta) = \frac{\sqrt{3}}{3}$: $\theta = \pi/6$

b. $\cos(\theta) = -\frac{1}{2}$ and $\sin(\theta) > 0$. : 2nd Quad: $\sin(\theta) = \frac{\sqrt{3}}{2}$ $\tan(\theta) = -\sqrt{3}$:

c. $\tan(\theta) = 1$ and $\cos(\theta) < 0$. ; 3rd Quad: $\sin(\theta) = -\frac{\sqrt{2}}{2}$ $\cos(\theta) = -\frac{\sqrt{2}}{2}$

d. $\sin(\theta) = -\frac{\sqrt{3}}{2}$ and $\cot(\theta) < 0$. ($\tan(\theta) < 0$) ; 4th Quad: $\cos(\theta) = \frac{1}{2}$ $\tan(\theta) = -\sqrt{3}$

e. $\tan(\theta) = -\sqrt{3}$ and $\cos(\theta) < 0$. : 2nd Quad: $\cos(\theta) = -\frac{1}{2}$ $\sin(\theta) = \frac{\sqrt{3}}{2}$

f. $\sec(\theta) = -2$ and $\sin(\theta) < 0$. : 3rd Quad: $\sin(\theta) = -\frac{\sqrt{3}}{2}$ $\tan(\theta) = \sqrt{3}$

g. $\cot(\theta) = \sqrt{3}$ and $\csc(\theta) > 0$. ($\sin\theta > 0$) ; 1st Quad: $\sin(\theta) = \frac{1}{2}$ $\cos(\theta) = \frac{\sqrt{3}}{2}$

$\cos(\theta) = -\frac{1}{2}$

$\tan(\theta) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

9. Toby also thinks the following trigonometric equations are true. Is he correct? Justify your answer.

a. $\sin\left(\pi - \frac{\pi}{3}\right) = \sin(\pi) - \sin\left(\frac{\pi}{3}\right)$

b. $\cos\left(\pi - \frac{\pi}{3}\right) = \cos(\pi) - \cos\left(\frac{\pi}{3}\right)$

c. $\tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{3}\right) - \tan\left(\frac{\pi}{6}\right)$

d. $\sin\left(\pi + \frac{\pi}{6}\right) = \sin(\pi) + \sin\left(\frac{\pi}{6}\right)$

e. $\cos\left(\pi + \frac{\pi}{4}\right) = \cos(\pi) + \cos\left(\frac{\pi}{4}\right)$

a. $\sin(\pi - \frac{\pi}{3}) = -\sin(-\frac{\pi}{3}) = \sin(\frac{\pi}{3})$

$\sin(\pi) - \sin(\frac{\pi}{3}) = 0 - \sin(\frac{\pi}{3})$

compare

a is not true

b. $\cos(\pi - \frac{\pi}{3}) = -\cos(-\frac{\pi}{3}) = -\cos(\pi/3)$

$\cos(\pi) - \cos(\frac{\pi}{3}) = -1 - \cos(\frac{\pi}{3})$

compare

b is not true

c. $\tan(\frac{\pi}{3} - \frac{\pi}{6}) = \tan(\frac{2\pi}{6} - \frac{\pi}{6}) = \tan(\frac{\pi}{6}) = \frac{\sqrt{3}}{3}$

$\tan(\frac{\pi}{3}) - \tan(\frac{\pi}{6}) = \sqrt{3} - \frac{\sqrt{3}}{3}$

compare

c is not true

d. $\sin(\pi + \frac{\pi}{6}) = -\sin(\frac{\pi}{6}) = -\frac{1}{2}$

$\sin(\pi) + \sin(\frac{\pi}{6}) = 0 + \frac{1}{2}$

Compare

d is not true.

e. $\cos(\pi + \frac{\pi}{4}) = -\cos(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$\cos(\pi) + \cos(\frac{\pi}{4}) = -1 + \frac{\sqrt{2}}{2}$

compare

e is not true