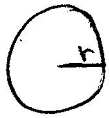
Related Rates HwK due 12/2 Short Answer #1-9

1. 

Given

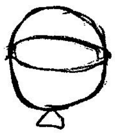
$\frac{dr}{dt} = 0.01 \frac{cm}{sec}$

WTK

$\frac{dA}{dt}$ when $r = 50$ cm?

Eqn: $A = \pi r^2$

deriv: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(50)(.01) = \boxed{1\pi \frac{cm^2}{sec}}$

2. 

Given

$\frac{dV}{dt} = 100\pi \frac{ft^3}{min}$

WTK

$\frac{dr}{dt}$ when $r = 5ft$? $\frac{dSA}{dt}$ when $r = 5ft$?

Eqn: $V = \frac{4}{3}\pi r^3$ (Volume)

Deriv: $\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

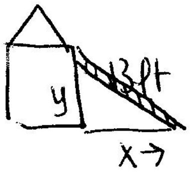
$100\pi = 4\pi(5)^2 \frac{dr}{dt}$

$\boxed{1 \frac{ft}{min}} = \frac{100\pi}{100\pi} = \frac{dr}{dt}$

Eqn: $SA = 4\pi r^2$ (surface area)

$\frac{dSA}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$\frac{dSA}{dt} = 8\pi(5)(1) = \boxed{40\pi \frac{ft^2}{min}}$

3. 

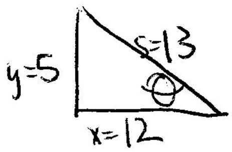
Given:

$\frac{dx}{dt} = 5 \frac{ft}{sec}$ when $x = 12$.

a) WTK: $\frac{dy}{dt}$ when $\frac{dx}{dt} = 5, x = 12$

Eqn: $x^2 + y^2 = 13^2$

Deriv: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$



$5^2 + 12^2 = 13^2$

$y = 5$

$y\frac{dy}{dt} = -x\frac{dx}{dt}$

$dy/dt = \frac{-x\, dx/dt}{y} = \frac{-12(5)}{5}$

notice the height "y" of the ladder is decreasing so dy/dt is negative.

$\boxed{\frac{dy}{dt} = -12 \text{ ft/sec}}$

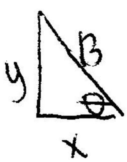
b. <u>WTK</u>: $\frac{dA}{dt}$ when $x=12$, $\frac{dx}{dt}=5$ $\frac{dy}{dt}=-12$, $y=5$

Eqn: $A=\frac{1}{2}xy$

Deriv: $\frac{dA}{dt}=\frac{1}{2}\left(x\,\frac{dy}{dt}+y\cdot\frac{dx}{dt}\right)=\frac{1}{2}\left(12(-12)+5(5)\right)=\frac{1}{2}(-144+25)$

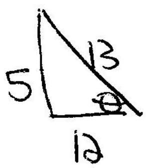
$\boxed{\frac{dA}{dt}=-\frac{119}{2}\ \frac{ft^2}{sec}}$ ← notice area of the Δ is also decreasing since $dA/dt<0$.

c. <u>WTK</u>: $\frac{d\theta}{dt}$ when $x=12$, $\frac{dx}{dt}=5$, $\frac{dy}{dt}=-12$, $y=5$



eqn: $\sin\theta = y/13$ $\quad 13\sin\theta = y$

deriv: $13\cos\theta\,\frac{d\theta}{dt}=\frac{dy}{dt}$

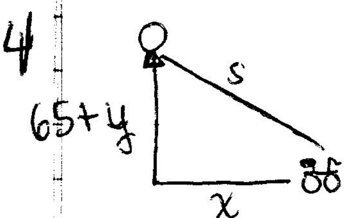


from Δ $\frac{adj}{hyp}$

$13\left(\frac{12}{13}\right)\cdot\frac{d\theta}{dt}=-12$

$\boxed{\frac{d\theta}{dt}=\frac{-12}{12}=-1\ \frac{rad}{sec}}$

4



(at t=0 balloon is already 65 ft off ground)

<u>Given</u>

$\frac{dy}{dt}=1\ ft/sec$; $\frac{dx}{dt}=17\ ft/sec$

<u>WTK</u>: $\frac{ds}{dt}$ after 3 seconds

eqn: $x^2+y^2=s^2$

diff: $\cancel{2}x\frac{dx}{dt}+\cancel{2}y\frac{dy}{dt}=\cancel{2}s\,\frac{ds}{dt}$

$51(17)+68(1)=85\ ds/dt$

at $t=3$

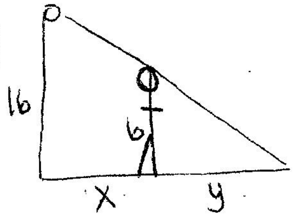
$x=3(17)=51$

$y=3(1)=3\ (+65)$

$s=\sqrt{51^2+68^2}=85$

$ds/dt=\frac{51(17)+68}{85}=\boxed{11\ ft/sec}$

5



WTF: $\frac{dy}{dt}$ when $x = 10$, $\frac{dx}{dt} = -5\text{ ft/sec}$

eqn: $\frac{y}{6} = \frac{y+x}{16}$

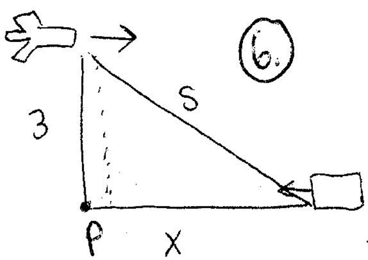
$$16y = 6y + 6x$$

$$10y = 6x$$

eqn: $y = \frac{3x}{5}$

$\frac{dy}{dt} = \frac{3}{5} \cdot \frac{dx}{dt}$ $\boxed{\frac{dy}{dt} = \frac{3}{5}(-5) = -3\text{ ft/sec}}$

6.



let p be the distance airplane has traved since seeing the car.

find $\frac{dx}{dt}$ when $\frac{dp}{dt} = 120\text{mph}$, $s = 5\text{mi}$, $\frac{ds}{dt} = -160\text{ mph}$

eqn: $s^2 = 3^2 + (x-p)^2$

diff: $2s \cdot \frac{ds}{dt} = 2(x-p)\left(\frac{dx}{dt} - \frac{dp}{dt}\right)$

$2(5)\cdot(-160) = 2(4-0)\left(\frac{dx}{dt} - 120\right)$

$-1600 = 8\frac{dx}{dt} - 960$

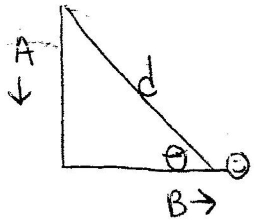
$\frac{-640}{8} = \frac{8\frac{dx}{dt}}{8}$ $\boxed{\frac{dx}{dt} = -80\text{ mph}}$

$s^2 = x^2 + 3^2$

$25 = x^2 + 9$

$x^2 = 16$ $x = 4$

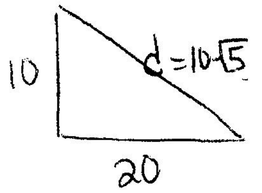
7



WTK: $\frac{d\theta}{dt}$ when $A = 10m$, $B = 20m$, $\frac{dA}{dt} = -2 \, m/sec$, $\frac{dB}{dt} = 1 \, m/sec$

eqn: $\tan\theta = \frac{A}{B}$

diff: $\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{B\frac{dA}{dt} - A \cdot \frac{dB}{dt}}{B^2}$



$d^2 = 10^2 + 20^2 = 100 + 400 = 500$

$d = \sqrt{500} = 10\sqrt{5}$

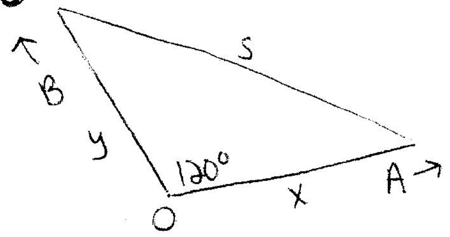
$\cos\theta = \frac{20}{10\sqrt{5}} = \frac{2}{\sqrt{5}}$ so $\sec\theta = \frac{\sqrt{5}}{2}$

sub in: $\left(\frac{\sqrt{5}}{2}\right)^2 \cdot \frac{d\theta}{dt} = \frac{20(-2) - 10(1)}{20^2}$

$\frac{5}{4} \cdot \frac{d\theta}{dt} = \frac{-40 - 10}{400}$

$\boxed{\frac{d\theta}{dt} = \frac{-50}{400}\left(\frac{4}{5}\right) = \frac{-.1 \text{ rad}}{\text{sec}}}$

8.



WTF: $\frac{ds}{dt}$ when $OA = 5$, $OB = 3$, $\frac{dx}{dt} = 14$, $\frac{dy}{dt} = 21$
($OA = x$, $OB = y$)

$s^2 = x^2 + y^2 - 2xy\cos(120°)$

$s^2 = x^2 + y^2 - 2xy(-\tfrac{1}{2})$

eqn: $s^2 = x^2 + y^2 + xy$

diff: $2s \cdot \frac{ds}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} + x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt}$

when $x = 5$, $y = 3$, $\theta = 120°$

$s^2 = 5^2 + 3^2 + (3)(5) = 25 + 9 + 15 = 49$

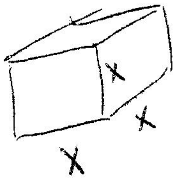
$s = 7$

$2(7) \cdot \frac{ds}{dt} = 2(5) \cdot (14) + 2(3) \cdot 21 + (5)(21) + 3(14)$

$14 \frac{ds}{dt} = 413$

$\boxed{\frac{ds}{dt} = 413/14 = 29.5 \text{ knots}}$

9.



Volume increasing: $\frac{dV}{dt} = 1200 \, cm^3/sec$ when $x = 20$

WTK: $\frac{dx}{dt}$

$V = x^3$

$\frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt}$

$1200 = 3(20^2) \frac{dx}{dt}$

$\boxed{1 \, \frac{cm}{sec}}\ \frac{1200}{3(400)} = \boxed{\frac{dx}{dt}}$