Markwalter BC Calc HW 4.5

Name:

Particle Motion and Graphs of Velocity

1) A particle moves along a horizontal line so that its position at any time $t \geq 0$ is given by the function $x(t) = t^2 - 4t + 3$, where x is measured in feet and t is measured in seconds.

(a) Find the displacement of the particle during the first 3 seconds. Explain its meaning.

$X(3) - X(0) = 0 - 3 = -3$ ft

The particle move threefeet left.

Average rate of change of position

(b) Find the average velocity of the particle during the first 3 seconds. Explain its meaning.

$\frac{X(3) - X(0)}{3 - 0} = \frac{-3}{3} = -1$ ft/s   On average, the particle moves -1 ft/s over the first 3 seconds.

(c) Find the particle's initial velocity and its velocity at $t = 3$ seconds. Explain the meanings of each in terms of the particle's movement.

$x'(t) = v(t) = 2t - 4$

$v(0) = -4$ ft/s   the particle is moving 4 ft/s to the left at 0 seconds

$v(3) = 2$ ft/s   the particle is moving 2 ft/s to right at 3 seconds.

(d) Find the acceleration of the particle when $t = 3$ seconds. Explain its meaning in terms of the particle's velocity.

$a(t) = 2$

$a(3) = 2$ ft/s^2 the acceleration of the particle is 2 ft/sec^2, so the particle's velocity is increasing at a rate of 2 ft/sec^2 at $t = 3$ sec.

(e) At $t = 3$ seconds, is the speed of the particle increasing or decreasing? Justify.

$v(3) = 2$ ft/s   Speed of the particle is increasing b/c $v(3)$ and $a(3)$ are both positive (have the same sign).

$a(3) = 2$ ft/s^2

(f) During what times is the particle moving to the right? Left? At what values of t does the particle change direction? Justify.

$v(t) = 2t - 4$

v(t): 0 — 2 +

2t − 4: − | +

The particle is moving left on $[0, 2)$ b/c $v(t)$ is negative. The particle is moving right on $(2, \infty)$ b/c $v(t)$ is positive. The particle changes direction at $t = 2$ sec b/c $v(t)$ changes sign.

(g) Find the total distance the particle travels during the first 3 seconds. Are you as exhausted as the particle?

$X(0) = 3$

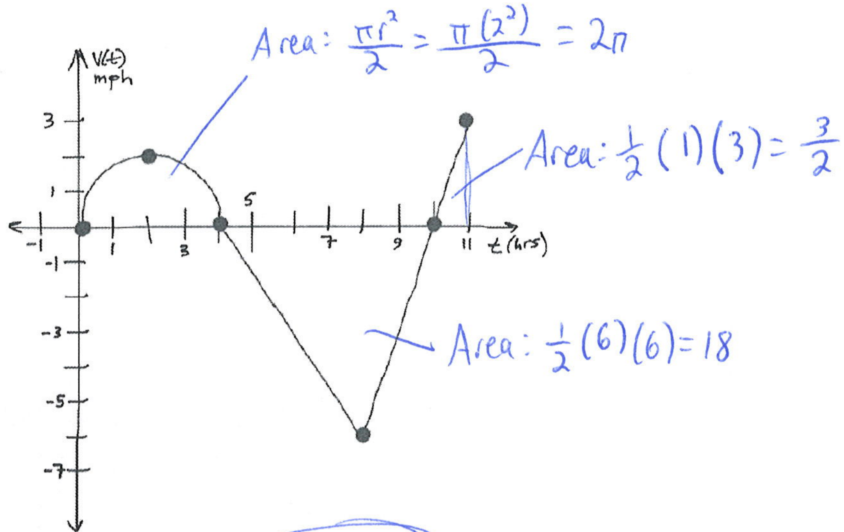
$x(2) = -1$ 4

$X(3) = 0$ 1

5 ft

No!

2)



The graph above shows the velocity, $v(t)$, in miles per hour of a particle moving along the x-axis for $0 \le t \le 11$ hours. It consists of a semi circle and two line segments. Use the graph and your knowledge of motion to answer the following questions.

(a) At what time, $0 \le t \le 11$ hours, is the speed of the particle the greatest?

Speed = $|v(t)|$ Speed is greatest at $t = 8$ hrs. $|v(t)| = 6$ mph at $t = 8$ hrs

(b) At which of the times, $t = 2$, $t = 6$, or $t = 9$ hours, is the acceleration of the particle greatest? Justify.

$v'(t) = a(t)$, so look at the slope of this graph $v'(2) = 0$ mph², $v'(6) = -\frac{6}{4}$ mph², $v'(9) = 3$ mph².
acceleration is greatest at $t = 9$ hr.

(c) Over what open time interval(s) $0 < t < 11$ hours is the particle moving to the left? Justify.

Particle is moving left when $v(t)$ is negative (below t-axis): $(4, 10)$.

(d) Over what open time interval(s) $0 < t < 11$ hours is the velocity of the particle increasing? Justify.

Velocity is increasing when $a(t)$ (the slope of $v(t)$) is positive: $(0, 2)$, $(8, 11)$

(e) Over what open time interval(s) $0 < t < 11$ hours is the speed of the particle increasing? Justify.

Speed is increasing when $a(t)$ and $v(t)$ have the same sign. Remember, $a(t)$ is the slope of $v(t)$: $(0,2)$, $(5,8)$, $(10,11)$

(f) At what times on $0 < t < 11$ is the acceleration of the particle undefined?

$a(t)$ is the slope or derivative of $v(t)$. $a(t)$ is undefined at $t = 4$ hr, $t = 8$ hrs.

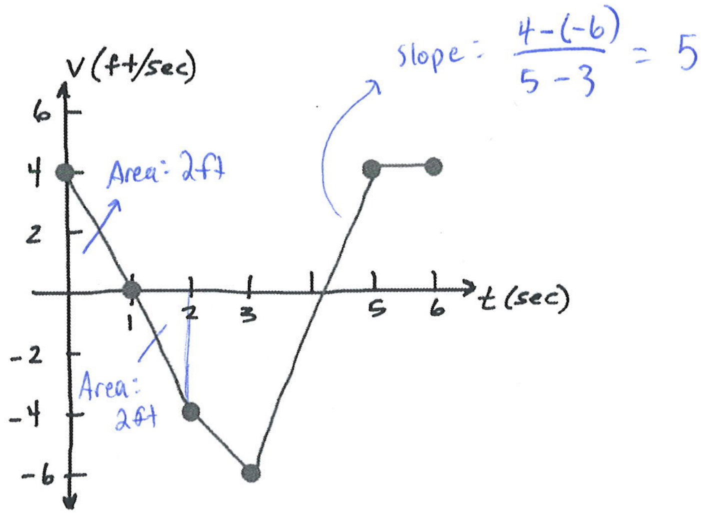
(g) Find the area of the semicircle on the interval $0 \le t \le 4$ bounded by the curve and the x-axis, then find the area of the triangle on the interval $4 \le t \le 10$ bounded by the curve and the x-axis, and finally, find the area of the triangle on the interval $10 \le t \le 11$ bounded by the curve and the x-axis. If all of these areas were positive and added together, propose what quantity this might be in terms of the particle's movement on $0 \le t \le 11$ hours.

Sum: $2\pi + 18 + \frac{3}{2}$

Units can help. E.g. Area $= \frac{1}{2}(6\text{ hrs})(6\frac{\text{mi}}{\text{hr}}) = 18$ mi

The sum of the positive areas would give the total distance traveled by the particle in miles from 0 to 11 hrs.

3)



The graph above shows the velocity $v(t)$ of a particle, in ft/sec, moving along a horizontal line for $0 \le t \le 6$ seconds.

(a) On what open intervals or at what time(s) $0 < t < 6$ is the particle at rest? Justify.

Particle is at rest when $v(t) = 0$: $t = 1$ sec, $t \approx 4.2$ sec

(b) On what open intervals $0 < t < 6$ is the particle moving to the right? Justify.

Particle is moving right when $v(t)$ is positive: $[0, 1)$, $(4.2, 6]$

(c) On what open intervals or at what time(s) $0 < t < 6$ is the particle moving at its greatest speed? Greatest velocity? → sign doesn't matter

Speed $= |v(t)|$, so greatest speed is at $t = 3$ sec when the speed is 6 ft/sec

Greatest velocity (now sign matters) is at $t = 0$ sec and $[5, 6]$ when velocity is 4 ft/sec

(d) On what open intervals or at what time(s) $0 < t < 6$ is the particle's speed increasing? Decreasing? Justify.

Speed is increasing when $v(t)$ and $a(t)$ have the same sign: $(1, 2), (2, 3), (4.2, 5)$

Speed is decreasing when $v(t)$ and $a(t)$ have different signs: $(0, 1), (3, 4.2)$

(e) What is the particle's acceleration at $t = 4.8$ second? Explain what this number means in terms of the particle's velocity.

acceleration is $v'(t)$: slope of $v(t)$! $v'(4.8) = 5$ ft/sec^2; the particle's velocity is increasing at a rate of 5 ft/sec^2

(f) On what open intervals or at what time(s) $0 < t < 6$ is the acceleration of the particle the greatest?

acceleration is greatest when $v'(t)$ (the slope of $v(t)$) is the greatest: $(3, 5)$ b/c $a(t)$ is 5 ft/s^2 there.

(g) (is for "genius") What is the particle's displacement during the first 2 seconds? Justify.

The particle's displacement is 0ft b/c the particle moves at the same rate right and left for the same amount of time OR The area under the velocity graph from $[0, 1]$ equals the area under the velocity graph from $[1, 2]$ but the signed areas are opposite. Thus the particle ends where it started!