Lesson Summary

The function $f(x) = \log_b(x)$ is defined for irrational and rational numbers. Its domain is all positive real numbers. Its range is all real numbers.

The function $f(x) = \log_b(x)$goes to negative infinity as x goes to zero. It goes to positive infinity as x goes to positive infinity.

The larger the base b, the more slowly the function $f(x) = \log_b(x)$ increases.

By the change of base formula, $\log_{\frac{1}{b}}(x) = -\log_b(x)$.

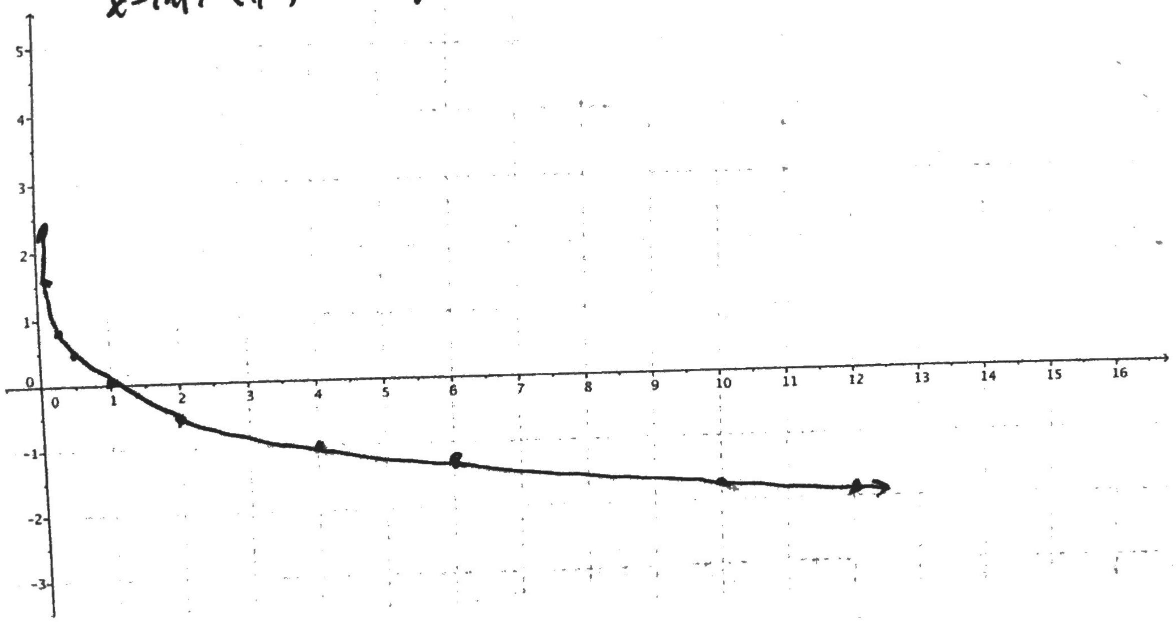
Problem Set

1. The function $Q(x) = \log_b(x)$ has function values in the table at right.
   a. Use the values in the table to sketch the graph of $y = Q(x)$.
   b. What is the value of b in $Q(x) = \log_b(x)$? Explain how you know.
   c. Identify the key features in the graph of $y = Q(x)$.

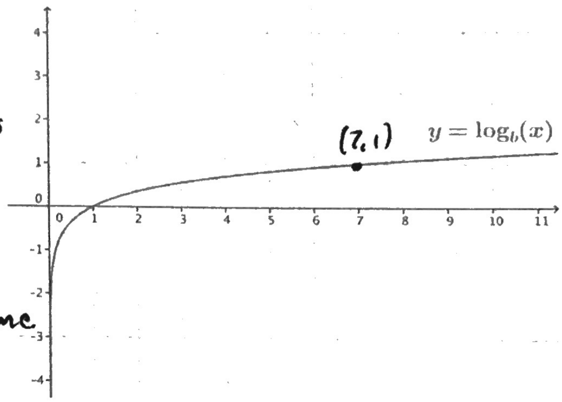
x	$Q(x)$
0.1	1.66
0.3	0.87
0.5	0.50
1.00	0.00
2.00	−0.50
4.00	−1.00
6.00	−1.29
10.00	−1.66
12.00	−1.79

b. The graph is reflected over x-axis so $\log_{\frac{1}{?}}(x)$
when $x=4$ $y=-1$ so $b=\frac{1}{4}$ $\log_{\frac{1}{4}}(x)$

c. y-int: none
x-int: (1,0)
Domain: $x>0$
Range: $(-\infty, \infty)$
As $x \to 0$ $y \to \infty$
As $x \to \infty$ $y \to -\infty$



Consider the logarithmic functions $f(x) = \log_b(x)$, $g(x) = \log_5(x)$, where b is a positive real number, and $b \neq 1$. The graph of f is given at right.



a. Is $b > 5$,or is $b < 5$? Explain how you know.

b>5 b/c the y-value of 1 occurs when x=7
So b is 7.

b. Compare the end behavior of f and g.

The end behaviors will be the same
As $x \to 0$, $y \to -\infty$
As $x \to \infty$, $y \to \infty$

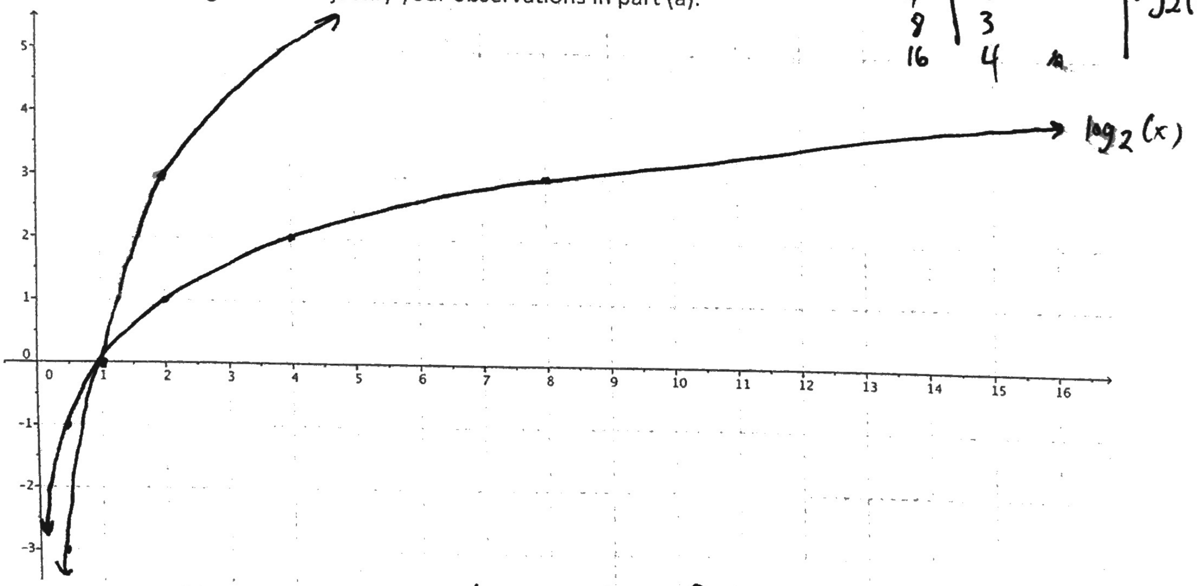
x	$f(x)$
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3
16	4

x	$g(x)$
1/2	$\log_2(\frac{1}{8}) = -3$
1	$\log_2(1) = 0$
2	$\log_2(8) = 3$
4	$\log_2(64)$

On the same set of axes, sketch the functions $f(x) = \log_2(x)$ and $g(x) = \log_2(x^3)$.

a. Describe a transformation that takes the graph of f to the graph of g.

b. Use properties of logarithms to justify your observations in part (a).



a. All of g(x)'s points are stretched vertically by a factor of 3.

b. $g(x) = \log_2(x^3) = 3\log_2(x) = 3f(x)$

scaled by a factor of 3.