Lesson 7: Solving Equations with Logarithms and Exponents

Classwork

Do Now

Work with your partner or group to solve each of the following equations for x.

a. $2^x = 2$ $x=1$	h. $\log_2(x) = \log_2(4)$ $x=4$
b. $2^x = 2^3$ $x=3$	i. $\log_3(2x) = \log_3(4)$ $x=2$
c. $2^x = 16$ $x=4$	j. $\log_8(3x+1) = \log_8(10)$ $3x+1=10$ $x=3$
d. $2^x - 64 = 0$ $x=6$	k. $\log_2(x) = \log_2(4) + \log_2(2)$ $x=8$
e. $2^x - 1 = 0$ $x=0$	
f. $2^{3x} = \frac{1}{2^6}$ $\quad 2^{3x}=2^{-6}$ $x=-2$	l. $\log_2(x^2) = \log_2(4)$ $x=\pm 2$
g. $2^{x+1} = 2^{2x-1}$ $x+1=2x-1$ $2=x$	m. $\log_{11}(x^2+3) = \log_{11}(-4x)$ $x^2+3=-4x$ $x^2+4x+3=0$ $(x+3)(x+1)=0$ $x=-3 \quad x=-1$

What happens when things mismatch?

In the real numbers, what are the solutions to the following equations.

Example:

1. $8^{2x+1} = 2^x$

$(2^3)^{2x+1} = 2^x$

$2^{6x+3} = 2^x$

$6x+3 = x$

$x = -\frac{3}{5}$

2. $9^{x+2} = 27^{x-1}$

$(3^2)^{x+2} = (3^3)^{x-1}$

$2x+4 = 3x-3$

$7 = x$

Practice:

1. $4^{-2x+1} = 16^{x+1}$

$(4)^{-2x+1} = 4^{2x+2}$

$-2x+1 = 2x+2$

$-4x = 1$

$x = -\frac{1}{4}$

2. $25^{2x+1} = 5^{x-4}$

$5^{4x+2} = 5^{x-4}$

$4x+2 = x-4$

$3x = -6$

$x = -2$

3. $2^{6x+3} = \frac{1}{4^x}$

$2^{6x+3} = 4^{-x}$

$2^{6x+3} = 2^{-2x}$

$6x+3 = -2x$

$8x = -3$

$x = -\frac{3}{8}$

4. $3^{-2x+2} = \frac{1}{9^{-x}}$

$3^{-2x+2} = 9^x$

$3^{-2x+2} = 3^{2x}$

$-2x+2 = 2x$

$2 = 4x$

$\frac{1}{2} = x$

Write the following logarithms in exponential form:

$\log_2(7) = x$

$2^x = 7$

$\log_5(17) = x$

$5^x = 17$

$\log_7(x) = 11$

$7^{11} = x$

Example: Solve the following equations by rewriting the logarithmic equation as an exponential one or vice versa.

1. ~~$\log_3(x) + 1 = 30$~~

$\log_3(x) + 1 = 1$

$\log_3(x) = 0$

$3^0 = x$

$x = 1$

2. $4\log_{12}(x) = 8$

$\log_{12}(x) = 2$

$12^2 = x$

$144 = x$

3. $2^{x+1} = 7$

$\log_2(7) = x+1$

$\log_2(7) - 1 = x$

4. $3^{2x} + 1 = 11$

$3^{2x} = 10$

$\log_3(10) = 2x$

$\frac{\log_3(10)}{2} = x$

So what? We can use <u>exponents</u> to get variables out of <u>logarithms</u>. And we can use <u>logarithms</u> to get variables out of <u>exponents</u>. They are <u>inverse</u> operations just like addition and subtraction or powers and roots.

Example: Solve the following equations by using inverse operations

1. $\log_4(2x) + 5 = 6$

$\log_4(2x) = 1$

$4^1 = 2x$

$4 = 2x$

$2 = x$

2. $3^{x+1} - 2 = 8$

$3^{x+1} = 10$

$\log_3(10) = x+1$

$\log_3(10) - 1 = x$

Practice: Solve the following equations by using inverse operations

a. $10^x = 3$

$\log_{10}(3) = x$

b. $7^{x^2+1} = 15$

$\log_7(15) = x^2+1$

$\log_7(15) - 1 = x^2$

$\pm\sqrt{\log_7(15) - 1} = x$

c. $5^{x-1} + 4 = 7$

$5^{x-1} = 3$

$\log_5(3) = x - 1$

$\log_5(3) + 1 = x$

d. $\log_3(x) - 7 = -5$

$\log_3(x) = 2$

$3^2 = x$

$9 = x$

e. $2\log_4(x) + 2 = 3$

$2\log_4(x) = 1$

$\log_4(x) = \frac{1}{2}$

$4^{1/2} = x$

$2 = x$

f. $\log_5(3x) + 7 = 9$

$\log_5(3x) = 2$

$5^2 = 3x$

$\frac{25}{3} = x$

Whiteboards

Problem Set

1. Solve each of the following equations for x using the same technique as was used in the Opening Exercise.

a. $2^x = 32$

$x = 5$

b. $2^{x-3} = 4^{2x+5}$

$2^{x-3} = (2^2)^{2x+5}$

$x - 3 = 4x + 10$

$-13 = 3x$

$\frac{-13}{3} = x$

c. $2^{x^2-3x} = 2^{-2}$

$x^2 - 3x = -2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2 \quad x = 1$

d. $2^x - 2^{4x-3} = 0$

$2^x = 2^{4x-3}$

$x = 4x - 3$

$3 = 3x$

$x = 1$

e. $2^{3x} \cdot 2^5 = 2^7$

$2^{3x+5} = 2^7$

$3x + 5 = 7$

$3x = 2$

$x = \frac{2}{3}$

f. $2^{x^2-16} = 1$

$2^{x^2-16} = 2^0$

$x^2 - 16 = 0$

$(x+4)(x-4) = 0$

$x = \pm 4$

g. $3^{2x} = 27$

$3^{2x} = 3^3$

$x = \frac{3}{2}$

h. $3^{\frac{2}{x}} = 81$

$3^{\frac{2}{x}} = 3^4$

$\frac{2}{x} = 4$

$2 = 4x$

$\frac{1}{2} = x$

i. $\frac{3^{x^2}}{3^{5x}} = 3^6$

$3^{x^2-5x} = 3^6$

$x^2 - 5x = 6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x = 6$

$x = -1$

2. Find consecutive integers that are under and over estimates of the solutions to the following exponential equations.

a. $2^x = 20$ x is between 4 and 5

b. $2^x = 100$ x is between 6 and 7

c. $3^x = 50$ x is between 3 and 4

3. Solve the following equations by using inverse operations

a. $3^{x+3} - 2 = 8$

$3^{x+3} = 10$

$\log_3(10) = x + 3$

$\log_3(10) - 3 = x$

b. $7^{2x} + 4 = 6$

$7^{2x} = 2$

$\log_7(2) = 2x$

$\frac{\log_7(2)}{2} = x$

c. $\log_4(2x) - 8 = -6$

$\log_4(2x) = 2$

$4^2 = 2x$

$16 = 2x$

$x = 8$

d. $\log_7(2x) = 1$

$7^1 = 2x$

$7 = 2x$

$\frac{7}{2} = x$

e. $2\log_4(3x) + 4 = 5$

$2\log_4(3x) = 1$

$\log_4(3x) = \frac{1}{2}$

$4^{1/2} = 3x$

$2 = 3x$

$\frac{2}{3} = x$

f. $-2\log_7(2x + 1) = -4$

$\log_7(2x+1) = 2$

$7^2 = 2x+1$

$49 = 2x+1$

$48 = 2x$

$24 = x$