Homework

2. Given the geometric series defined by the following values of a_0 and r, first find the explicit formula. Then find the value of n so that a_n has the specified value.

   a. $a_0 = 64, r = \frac{1}{2}, a_n = 2$

   64, 32, 16, 8, 4, 2
   0, 1, 2, 3, 4, 5

   $n=5$

   b. $a_0 = 13, r = 3, a_n = 85293$

   13, 39, 117, 351, 1053, 3159, 9477, 28431, 85293
   0, 1, 2, 3, 4, 5, 6, 7, 8

   $n=8$

   c. $a_0 = 6.7, r = 1.9, a_n = 7804.8$

   6.7, 12.73, 24.187, 45.96, 87.32, 165.9, 315.2, 598.9, 1137.9, 2162, 4107.8, 7804.8
   0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

   $n=11$

   d. $a_0 = 10958, r = 0.7, a_n = 25.5$

   10958, 7670.6, 5369.4, 3758.6, 2631, 1841.7, 1289.2, 902.4, 631.7, 442.14, 309.5,
   0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
   216.7, 151.7, 106.2, 74.3, 52.02, 36.4, 25.5
   11, 12, 13, 14, 15, 16, 17

   $n=17$

3. If a geometric sequence has $a_1 = 256$ and $a_8 = 512$, find the exact value of the common ratio r.

   $a_1 = 256$
   $a_8 = 512$

   256 ___ ___ ___ ___ ___ ___ 512
   1 8

   multiply by r 7 times

   $256r^7 = 512$
   $r^7 = 2$
   $r = \sqrt[7]{2}$

4. If a geometric sequence has $a_2 = 495$ and $a_6 = 311$, approximate the value of the common ratio r to four decimal places.

   495 ___ ___ ___ 311
   2 6

   $495r^4 = 311$

   $r^4 = \frac{311}{495}$

   $r = \sqrt[4]{\frac{311}{495}}$

5. A bouncy ball rebounds to 90% of the height of the preceding bounce. Craig drops a bouncy ball from a height of 20 ft.

a. Write out the sequence of the heights h_1, h_2, h_3, and h_4 of the first four bounces, counting the initial height as $h_0 = 20$.

$h_0 = 20$ ft
$h_1 = 20(0.9) = 18$ ft
$h_2 = 18(0.9) = 16.2$ ft
$h_3 = 14.58$ ft
$h_4 = 13.122$ ft

b. Write a recursive formula for the rebound height of a bouncy ball dropped from an initial height of 20 ft.

$a_n = a_{n-1}(0.9)$
multiply previous term by 0.9.

c. Write an explicit formula for the rebound height of a bouncy ball dropped from an initial height of 20 ft.

$A(n) = 20(0.9)^n$ no $n-1$ b/c the first term is $n=0$

d. How many bounces will it take until the rebound height is under 6 ft.?

$h_5 = 11.81$ ft
$h_6 = 10.62$ ft
$h_7 = 9.57$ ft
$h_8 = 8.61$ ft
$h_9 = 7.78$ ft
$h_{10} = 6.97$ ft
$h_{11} = 6.28$ ft
$h_{12} = 5.65$ ft
12 bounces

e. Extension: Find a formula for the minimum number of bounces needed for the rebound height to be under y ft., for a real number $0 < y < 20$.

$A(n) = 20(0.9)^n$
$y > 20(0.9)^n$
$\frac{y}{20} > 0.9^n$
$\log_{0.9}\left(\frac{y}{20}\right) > n$ → not done this yet.