Lesson 13: Exponential Problems Practice

1. The half-life of Plutonium-238 is 87.7 years. Thus, after 87.7 years, only half of any sample of Plutonium-238 will remain, and the rest will have decayed. We will assume that radioactive decay is modeled by exponential decay with a constant decay rate.

   a. Suppose we have a sample of 800 g of Plutonium-238. Write an exponential formula that gives the amount of Plutonium-238 remaining after m half-lives.

   $P(t)=800\left(\frac{1}{2}\right)^m$

   b. Suppose we have a sample of 800 g of Plutonium-238. Write an exponential formula that gives the amount of Plutonium-238 remaining after t years.

   $P(t)=800(0.5)^{t/87.7}$

   c. What is the decay rate per year?

   $P(t)=800(0.992)^t$

   d. How much Plutonium will remain after 100 years?

   $P(100)=800(0.5)^{100/87.7}=362.94$

   e. How long will it take before only 300g of Plutonium remain?

   $300=800(0.5)^{t/87.7}$

   $\frac{3}{8}=(0.5)^{t/87.7}$

   $\log_{(1/2)}(3/8)=t/87.7$

   $87.7\left(\log_{\frac{1}{2}}(3/8)\right)=t=\boxed{124.099\text{yrs}}$

2. You decide to invest your money in a bank that uses continuous compounding at 0.5% interest per year. You have $10000.

   a. How much money would you have in your account after 10 years? After 20 years? After 30 years? Is the amount of money increasing faster or slower?

$$A(t) = 10000e^{0.005t}$$

539

$A(10) = \$10512.71$

$A(20) = \$11051.71$ increasing

566.63

$A(30) = \$11618.34$

   b. If you invest all $10000 in the bank, how long will it take before you have $20000 in your account? Is this the best way to increase your wealth?

$$20000 = 10000e^{0.005t}$$

$$2 = e^{0.005t}$$

$$\ln(2) = 0.005t$$

$$\frac{\ln(2)}{0.005} = t = 138.63 \text{ yrs}$$

not the best way to build wealth.

The bacteria in a Petri dish culture are self-duplicating at a rapid pace.

The relationship between the elapsed time t, in minutes, and the number of bacteria, $B(t)$, in the Petri dish is modeled by the following function.

$$B(t) = 10 \cdot 2^{\frac{t}{12}}$$

$B(120) = 10 \cdot 2^{120/12} = 10 \cdot 2^{10} = 10{,}240$ bacteria

How many bacteria will make up the culture after 120 minutes?

Round your answer, if necessary, to the nearest hundredth.

A pot of piping hot stew has been removed from the stove and left to cool.

The relationship between the elapsed time, m, in minutes, since the stew was removed from the stove, and the temperature of the stew, $T(m)$, measured in $^\circ C$, is modeled by the following function.

$$T(m) = 20 + 50 \cdot 10^{-0.04m}$$

t=?

How many minutes will it take for the stew to cool to a temperature of $30^\circ C$?
Round your answer, if necessary, to the nearest hundredth.

$30 = 20 + 50 \cdot 10^{-0.04m}$

$10 = 50 \cdot 10^{-0.04m}$

$\frac{1}{5} = 10^{-0.04m}$

$\log(\frac{1}{5}) = -0.04m$

$m = \frac{\log(\frac{1}{5})}{-0.04} = 17.47$ minutes

Ella deposited $\$2500$ into a savings account.

The relationship between the time, t, in years, since the account was first opened, and Ella's account balance, $B(t)$, in dollars, is modeled by the following function.

$$B(t) = 2500 \cdot e^{0.025t}$$

What will the balance of Ella's savings account be after 4 years?
Round your answer, if necessary, to the nearest hundredth.

$B(4) = 2500\, e^{0.025(4)} = \$2762.93$

A large brine tank containing a solution of salt and water is being diluted with fresh water.

The relationship between the elapsed time, t, in hours, after the dilution begins, and the concentration of salt in the tank, $S(t)$, in grams per liter (g/l), is modeled by the following function.

$$S(t) = 600 \cdot e^{-0.3t}$$

t=?

How many hours will it take for the concentration of salt to decrease to 100 g/l?
Round your answer, if necessary, to the nearest hundredth.

$100 = 600e^{-0.3t}$

$\frac{1}{6} = e^{-0.3t}$

$\ln(\frac{1}{6}) = -0.3t$

$t = \frac{\ln(\frac{1}{6})}{-0.3} = 5.97$ hrs